VOLUME LXIV NUMBER 2

THE ASTROPHYSICAL JOURNAL

AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

Edited by

GEORGE E. HALE
Mount Wilson Observatory of the Carnegie Institution of Washington

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SEPTEMBER 1926

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THE UNIVERSITY OF CHICAGO PRESS
CHICAGO, ILLINOIS, U.S.A.

THE CAMBRIDGE UNIVERSITY PRESS, London
THE MARUZEN-KABUSHIKI-KAISHA, Tokyo, Osaka, Kyoto, Fukuoka, Sendai
THE COMMERCIAL PRESS, LIMITED, Shanghai

September 1926 Vol. LXIV, No. 2

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The following are authorized to quote the prices indicated:

For the British Empire: The Cambridge University Press, Fetter Lane, London, E.C. 4. Yearly subscriptions, including postage, £1 12s. 6d. each; single copies, including postage, 4s each.

For China: The Commercial Press, Ltd., Pacahon Road, Shanghai. Yearly subscriptions, $6.00; single copies, 75 cents, or their equivalents in Chinese money. Postage extra, on yearly subscriptions 30 cents, on single copies 5 cents.

Claims for missing numbers should be made within the month following the regular month of publication. The publishers expect to supply missing numbers free only when losses have been sustained in transit, and when the reserve stock will permit.

Business Correspondence should be addressed to The University of Chicago Press, Chicago, Illinois.

Communications for the editors and manuscripts should be addressed to the Editors of THE ASTROPHYSICAL JOURNAL, Yerkes Observatory, Williams Bay, Wisconsin.

The cable address is "University, Chicago."

The articles in this Journal are indexed in the *International Index to Periodicals*, New York, N.Y.

Entered as second-class matter, January 17, 1895, at the Postoffice at Chicago, Ill., under the Act of March 3, 1879.
Acceptance for mailing at special rate of postage provided for in Section 1103, Act of October, 3, 1917, authorized on July 13, 1918.

PRINTED IN THE U.S.A.

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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

VOLUME LXIV SEPTEMBER 1926 NUMBER 2

THE FINE STRUCTURE AND THE WAVE-LENGTHS OF THE BALMER LINES

By WILLIAM V. HOUSTON[1]

ABSTRACT

A Fabry-Perot interferometer was used to eliminate distortion by neighboring components, and to measure absolute wave-lengths. The first three lines of the Balmer series in hydrogen at the temperature of liquid air have been studied.

Each line is a doublet with the differences of wave-number 0.315 for $H\alpha$, 0.331 for $H\beta$, and 0.353 for $H\gamma$. The *absolute wave-lengths* are 6562.852 and 6562.716 for $H\alpha$, 4861.362 and 4861.284 for $H\beta$, and 4340.497 and 4340.429 for $H\gamma$. These values give the *Rydberg constant* as 109677.70 ± 0.04.

The doublet separation decreases and the intensity of the component of short wave-length increases with an increase of current. The doublet separation in the light from the end of the tube is greater than in that from the side when the current is high enough. The light from the side shows the corresponding polarization.

The observations may be *explained* by assuming that those *forbidden components* for which $\Delta k = 0$ *are present*, are *polarized* with the electric *vector parallel* to the tube, and increase in intensity with an increase of current.

The lines of the Balmer series are of especial interest because they form one of the two groups which have been given a complete theoretical treatment. Although a very large amount of experimental work has been done upon them, and nearly thirty papers have been published, there is as yet no very general agreement concerning their structure. Extensive references to the literature have been given by G. M. Shrum[2] and G. Hansen,[3] while E. Lau[4] has given a review of

[1] National Research Fellow in Physics.

[2] *Proceedings of the Royal Society*, A, **105**, 259, 1924.

[3] *Annalen der Physik*, **78**, 558, 1925.

[4] *Physikalische Zeitschrift*, **25**, 60, 1924.

the work up to 1924. Hence these need not be repeated here. The present paper is concerned with results which, it is believed, represent an increase in precision over the previous work.

Source of radiation.—It is necessary to use a source which will produce the Balmer lines with the greatest possible intensity relative to the secondary spectrum, and which will give the narrowest possible lines. The first requirement was met by the use of water-vapor as discovered by R. W. Wood,[1] while the second was attained by cooling the discharge tube in liquid air.

The discharge tube was 50 cm long and 5 mm in diameter, and was bent so that it could be inserted in a flask of liquid air. Light could be taken either from the end of a section of it, or from the side of a short horizontal portion. The hydrogen was obtained from a tank and bubbled through water, without any purification. The pump was operated continuously and the rate of flow of the hydrogen was controlled by a pinch cock on a rubber tube.

Excitation was by means of a Thordarson 1-kw transformer which gave 50 ma with a potential drop across the tube of about 2500 volts. For heavier currents a 10-kw General Electric transformer was used which gave up to 300 ma with about the same potential drop across the tube. The tube was operated for 5 seconds and then cooled for 15 seconds, so that its temperature did not rise much above that of the surrounding liquid air.

Optical apparatus.—The most common instrument used to study these lines has been the Lummer-Gehrcke plate. This, however, has several disadvantages.[2] Ordinarily the two lines of the doublet are separated by less than half the distance between adjacent orders. If the measurement is made visually with a comparator, this almost invariably leads to too high a value for the separation. On the other hand, if a microphotometer is used, a correction must be made in the opposite direction for the effect of one component on the other.

To avoid these disadvantages, and to make possible the measurement of absolute wave-lengths, the Fabry-Perot interferometer was used for this investigation. This permits the dispersion to be so adjusted that one component falls halfway between adjacent orders

[1] *Philosophical Magazine*, **42**, 729, 1921.

[2] P. H. van Cittert, *Annalen der Physik*, **77**, 372, 1925; see also Hansen, *loc. cit.*

of the other, and so to a large extent avoids the errors mentioned above.

The instrument used was constructed in the laboratory under the direction of Mr. Julius Pearson. The adjustment of the plates is made by means of differential screws which are both sensitive and permanent. The light is passed through a collimator with a wide slit, then to the interferometer, and through a constant-deviation prism to the camera. This arrangement is economical of light, but because of the moderate dispersion of the prism cannot be used on very complicated spectra. The exposure-time required was about 15 minutes, during which the temperature remained constant within about 0.05 degree.

Calculations.—The fractional order of interference at the center of the fringe system was found by the usual relation

$$\delta_n = \frac{x_n^2}{x_n^2 - x_{n-1}^2} - n ,$$

where x_n is the linear diameter of the nth fringe from the center of the system. This equation is true as long as the cosine of the angular radius of the fringe can be represented by the first two terms of its series. The angular diameter of the largest fringe measured was about 0.05 radian, so that the requirement is fulfilled.

For measurements of frequency-differences another property of the interferometer is convenient. If λ' and λ'' are the wave-lengths of two lines whose fringes have the fractional orders of interference δ' and δ'', then

$$(p+\delta')\lambda' = (p+\delta'')\lambda'' ,$$

where p is the common integral order of interference. Then

$$\delta'\lambda' - \delta''\lambda'' = p\Delta\lambda = \lambda(\delta' - \delta'') ,$$

where λ represents either wave-length, since they are nearly the same. Then

$$\Delta\nu = \frac{\Delta\lambda}{\lambda^2} = \frac{1}{p\lambda}(\delta' - \delta'') = \frac{1}{2d}(\delta' - \delta'') .$$

If, then, $\Delta\nu_0$ is the difference in frequency corresponding to the distance between adjacent orders, we have $\Delta\nu_0 = \frac{1}{2}d$, which is independent of the wave-length. Thus, if we have a series of doublets with a constant frequency-difference, it is possible to make $(\delta' - \delta'') = \frac{1}{2}$ for all of them at the same time.

The standards of wave-length used were the two yellow lines of mercury. The wave-length of one was taken from the work of Fabry and Perot and corrected to the international unit, which gave 5769.5970 I.A. The observed maximum of the other was disturbed by the presence of a satellite. For this reason it was measured, at the interferometer separation used, with reference to five helium lines. This gave the value 5790.6614 I.A., which was used for its wave-length. To avoid the possibility of a relative shift, half of the hydrogen exposure was made, the mercury lines were reflected in, and then the hydrogen exposure was finished.

Through the kindness of Dr. Pettit, of the Mount Wilson Observatory, microphotometer curves were made of the plates used for the observations. This permitted a precision not otherwise attainable.

Results.—The results may be divided into two classes: first, the determination of the structure of the lines in their normal or undisturbed condition; second, the effect of changing the conditions of excitation.

The plates used as representing normal conditions were taken when the pressure in the discharge tube was 0.5 mm and the current was 50 ma. The current density was thus 0.25 ampere per square centimeter. These conditions are probably suitable since Hansen, using lower pressures and currents, obtained the same separation of $H\alpha$, the most sensitive line.

Figure 1*a* shows a plate taken under normal conditions. It can be seen that the fraction of $\Delta\nu_0$ represented by $\Delta\nu$ increases steadily from $H\alpha$ to $H\gamma$. This is conclusive evidence that the frequency-difference increases in these doublets. $\Delta\nu_0 = 0.6586$ cm^{-1} in the plate shown.

Table I shows the values obtained from the measurement of two plates taken under these conditions. The first of these used the light from the side of the tube, and the other used that from the end. There is no certain difference between these two, although the light

from the end shows a slightly larger separation for all lines. The value for $H\gamma$ is subject to correction on both plates because of the fact that $\Delta\nu$ is greater than half of $\Delta\nu_0$. This causes a slight dis-

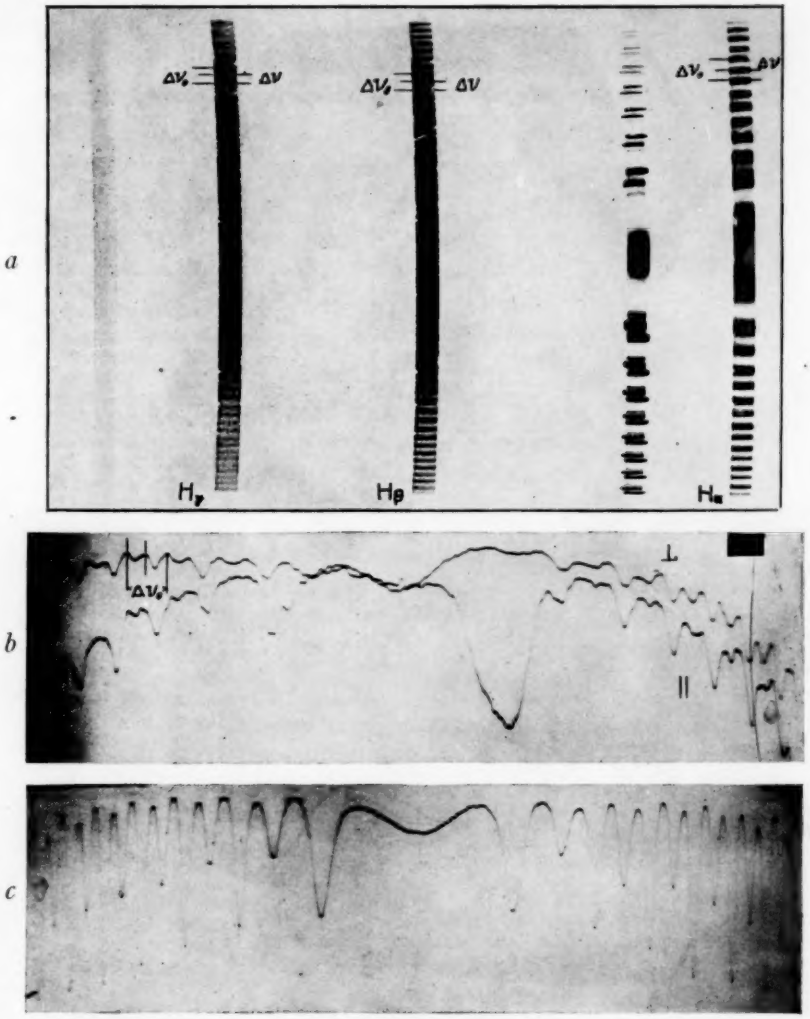

FIG. 1

a—A plate taken under normal conditions
b—Effect of polarization
c—Curve for $H\alpha$

placement of the maximum due to the overlapping of the next order. The amount of the correction is taken from a table given by Hansen.[1]

These values for $H\alpha$ are not widely different from others recently obtained. McLennan and Ireton[2] find .316 for $H\alpha$ and .343 for $H\beta$, while Hansen finds .316 for $H\alpha$ but only slightly larger values for $H\beta$ and $H\gamma$. Shrum's photographs as remeasured by Janicki[3] give .302 for $H\alpha$, while Janicki's[4] value from the plates of Gehrcke and Lau is .306. The increase in separation clearly shown in Figure 1

TABLE I

DOUBLET SEPARATIONS

Line	6563	4861	4340
No. of values	11	15	17
Mean of Plate 55	.3143	.3300	.3574
Mean of Plate 56	.3165	.3321	.3633
Mean of both	.3154	.3310	.3605
Mean deviation	.0024	.0051	.0073
Correction			.0073
Probable value:			
$\Delta\nu$	.3154 ± .002	.3310 ± .005	.3532 ± .007
$\Delta\lambda$	.1358	.0782	.0665

then requires that $H\beta$ and $H\gamma$ be wider than the values given by these observers of .317, .322, and .306, respectively, for $H\beta$, and .328, .318, and .315 for $H\gamma$.

Table II shows the values of the wave-lengths at 15° C. and 760-mm pressure, and the corresponding wave-numbers reduced to vacuum. The values given by W. E. Curtis[5] for the unresolved lines are close to the centers of gravity.

When the current-density is high enough, there is a difference in the apparent doublet separation between the light polarized parallel to the tube and that polarized perpendicular to the tube. Figure 1*b* gives the microphotometer curves taken from a plate which shows this effect. The current in the tube was 300 ma, making a current density of 1.50 amperes per square centimeter. For the upper curve, the electric vector was perpendicular; for the lower,

[1] *Loc. cit.* [2] *Proceedings of the Royal Society of Canada*, **19**, 77, 1925.

[3] *Annalen der Physik*, **76**, 561, 1925.

[4] Janicki and Lau, *Zeitschrift für Physik*, **35**, 1, 1925.

[5] *Proceedings of the Royal Society of London*, A, **90**, 605, 1914.

parallel to the tube. The lines are broadened by the heavy current but the difference in separation can be clearly seen. The same difference is seen between the light from the end and that from the side of the tube when the current-density is as high as this. A current-

TABLE II

ABSOLUTE WAVE-LENGTHS

Line	Wave-Length in I.A	Wave-Number	I_s/I_l
Hα.....	6562.852 ± .002 6562.716 ± .002	15233.078 ± .005 15233.395 ± .005	.80
Hβ.....	4861.362 ± .002 4861.284 ± .002	20564.640 ± .007 20564.971 ± .007	.94
Hγ.....	4340.497 ± .002 4340.429 ± .002	23032.385 ± .008 23032.745 ± .008	.98

density of 0.67 ampere per square centimeter does not produce an unmistakable effect.

Table III shows the effect of the current-density on the apparent separation of the lines. The doublet separation decreases with an increase in current-density. Hansen finds no such effect, but his current-densities are much lower than these.

TABLE III

EFFECT OF CURRENT-DENSITY ON DOUBLET SEPARATION

CURRENT IN AMP./SQ. CM	6563		4861	
	Side	End	Side	End
0.25.....	.314	.316	.331	.332
0.67.....	.311	.....	.326	.....
1.50.....	.29	.309	.....	.....

Table IV shows the effect of current-density upon the relative intensity of the components. The values are only approximate, but the component of short wave-length clearly increases in relative intensity with an increase in current. Hansen finds the same effect for a decrease in pressure.

Summary of experimental results.—(1) The doublet separation increases from 0.315 to 0.353 cm^{-1} in going from *Hα* to *Hγ*. (2) The

absolute wave-lengths of both components of the first three lines of the series are given. (3) The apparent doublet separation decreases with an increase in current density, while the component of short wave-length increases in relative intensity. (4) At a high-enough current-density the light polarized with its electric vector perpendicular to the tube shows a wider doublet separation than the light with the electric vector parallel to the tube.

Discussion of results.—Figure 2 shows the theoretical structure of these lines on the basis of Sommerfeld's theory. The full lines represent the components permitted by the selection principle,

TABLE IV

EFFECT OF CURRENT-DENSITY ON INTENSITY RATIO

CURRENT IN AMP./SQ. CM	I_s/I_l	
	6563	4861
0.25	.80	.94
0.67	.84	.92
1.50	1.05	1.02

while the dotted lines represent those forbidden because the jump in the azimuthal quantum number is greater than one. The crossed lines represent those forbidden because they correspond to no change in the azimuthal quantum number. The first line beneath each drawing represents the distance between the centers of gravity when this selection principle is used. The assumption is made that the two components in the group of long wave-length have the intensity ratio 10:1. The ratio is probably greater than this. In no case, however, could the apparent separation be reduced below 0.329 for $H\alpha$. Thus it is clear that the observed values of the doublet separation do not agree with the simple theory.

However, the observations can be explained by assuming that the components represented by the crossed lines actually are present, increase in intensity with an increase in current, and are polarized with the electric vector parallel to the direction of the current.

Hansen observed a distortion of the intensity-curve for the more refrangible component of the doublet, which he interpreted as being caused by the presence of one of these forbidden components.

Figure 1*c* shows a microphotometer curve taken of *Hα* on which there is no certain deviation from symmetry. The polarization phenomena are, however, even more conclusive evidence that the components of the doublet are not simple lines but are groups of lines, whose positions agree qualitatively with the theory.

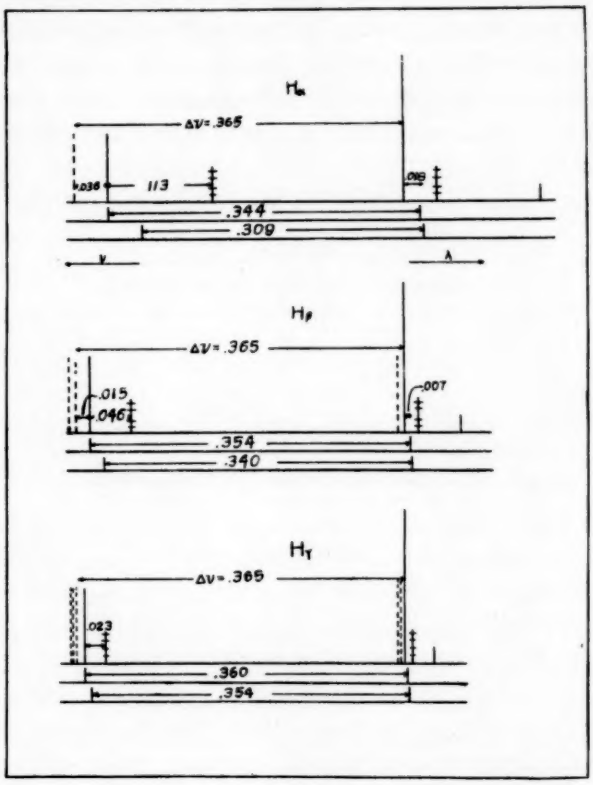

FIG. 2.—Theoretical structure

According to the correspondence principle, these lines may be produced by a disturbing electric field, which may be the field in the discharge tube. For this case Kramers has calculated the relative intensities of the components, but his values are much too small to satisfy the observations.

According to a letter from Professor Ehrenfest to Professor Millikan, Uhlenbeck and Goudsmit at Leiden have applied A. H.

Compton's suggestion of spinning electrons to spectral theory. According to them, the magnetic effect of the spinning electron is superposed upon the relativity effect of the orbit. This, they say, gives the same levels as before, but with a different assignment of quantum numbers. The inner quantum numbers are found to be the same as the azimuthal quantum numbers assigned by Sommerfeld. Hence the principle of choice must be that for inner quantum numbers, and the lines crossed in the figure are always permitted. This explains the observed doublet separation very nicely, but, until more is known about the polarization and relative intensities required by the theory, it is impossible to judge to what extent the observations confirm it. It may be mentioned that all the work on

TABLE V

SCHEME OF EXPECTED INTENSITIES

J	1	2	3	
1.....	22	45	.....	67
2.....	11	22	100	133
Total.	33	67	100	

the lines of ionized helium has shown the presence of the forbidden components in question.

Table V gives the relative intensities to be expected on this theory, if each level is assigned a quantum weight equal to its inner quantum number. This is the ordinary procedure. It is, of course, not sound in this case since each level is composed of two coinciding levels and should be weighted accordingly. Furthermore, the rule, as ordinarily employed, applies only to inner quantum numbers with the same azimuthal quantum number. However, it may be used as a suggestion until more is known about the proper method of treatment.[1]

The second line under each drawing in Figure 2 shows the centers of gravity based upon these intensities, and they agree fairly well with the observations. A slight change in the assumed relative

[1] Since this was written the intensities have been treated more fully. See Sommerfeld and Unsöld, *Zeitschrift für Physik*, **35**, 259, 1926, and Houston, *Bulletin of the American Physical Society*, **1**, No. 10, p. 9.

intensities will make the difference. Then, too, the assumed intensities cannot be correct because they give a constant ratio between the two groups for all lines, while the observed ratio increases.

Determination of "R."—From the values of the frequencies the Rydberg constant may be determined. The first column of Table VI shows the observed values of the wave-numbers for the groups of long wave-length. The second gives these corrected to the strongest line upon the assumption of the intensities in Table V. The third

TABLE VI

OBSERVED AND CALCULATED WAVE-NUMBERS

OBSERVED		CALCULATED		
Center of Gr.	Cor.	Relativity		Balmer
15233.078.....	.096	.079	.087	.078
20564.640.....	.647	.635	.647	.655
23032.385.....	.389	.387	.400	.413
	$R=109677.64$		.70	8.16

Best value of $R=109677.70\pm.04$.

column contains the values calculated from Sommerfeld's equation with the value of R given at the bottom. These values agree exceedingly well with the observed centers of gravity, but not quite so well with the corrected values. This indicates a change in the assumed intensities in the same way as that indicated by the doublet separations. The fourth column shows the values calculated with another value of R, while the last gives the result of applying a simple Balmer formula. It is clear from this that a Balmer formula cannot be made to fit by adjustment of the constant, and that the relativity correction is necessary. The best value for R is seen to be 109677.70 ± 0.04, in exact agreement with Birge.[1]

Conclusions.—The observations may be explained by assuming that those forbidden components for which $\Delta k=0$ are present, are polarized with the electric vector parallel to the tube, and increase in intensity with an increase of current.

[1] *Physical Review*, **17**, 589, 1921.

Thus the simple Sommerfeld theory requires a modification, and the modification by means of Kramers' application of the correspondence principle is not quantitatively sufficient. Whether the modification due to Uhlenbeck and Goudsmit is sufficient depends upon the relative intensities and polarizations required by their theory. The data available is sufficient for a test of the theory on these points.

This work was begun at the Ohio State University under the direction of Professor A. D. Cole, and has been repeated here with revised apparatus. I am indebted to both institutions for the equipment placed at my disposal.

NORMAN BRIDGE LABORATORY OF PHYSICS
March 10, 1926

SOLAR HYDRODYNAMICS[1]

By V. BJERKNES[2]

ABSTRACT

Hydrodynamical discussion of internal solar motions, especially vortex motions for explaining sun-spots.—It is shown that the *low temperatures* of sun-spots may be explained if the vortices connected with them decrease in intensity from the surface downward. The excess of centrifugal force in the upper part of the vortex then gives rise to a centrifugal pumping effect, by means of which the gases in the central region are lifted from below and adiabatically cooled.

A simple view of the numerous collective *properties of sun-spots* is obtained if we make the following suppositions: Sun-spot phenomena originate in two sub-photospheric zonal vortices, one for each cycle, which have opposite rotations. These vortices occasionally rise to the surface and penetrate the sun's atmosphere, and then return again below the photosphere. Wherever a sinuous vortex rises and cuts the photosphere a sun-spot of the corresponding cycle makes its appearance or may be discovered. The direction of rotation is supposed to determine the sign of the magnetic field. In virtue of the internal circulation of the sun, which may also explain the phenomenon of the equatorial acceleration, the two vortices revolve round each other, the period of revolution being the complete sun-spot period of 22 years.

Hale's discovery of magnetic fields in the sun-spots, of the regular succession of the polarities in binary groups, and of the reversal of these polarities from cycle to cycle has created an entirely new situation as regards the theory of sun-spots and of solar phenomena in general.

No attempt has yet been made, as far as I know, to give a connected interpretation of these phenomena. Any interpretation, however improbable, may therefore have a certain relative value. But in taking up the subject I must emphasize the fact that I enter a field entirely new to me. All my knowledge of solar phenomena has resulted from the recent opportunity for discussion with the astronomers of the Mount Wilson Observatory, to whom I here express my appreciative thanks for their interest and assistance.

I. RESULTS AND PRINCIPLES OF PHYSICAL HYDRODYNAMICS

We shall begin by summarizing certain general principles and results which will be used later, and then add a number of supplementary developments.[3]

[1] *Contributions from the Mount Wilson Observatory*, No. 312.

[2] Research Associate of the Carnegie Institution of Washington.

[3] Cf. the lectures given at the California Institute of Technology, Pasadena, September to December, 1924, which will be published later. The concluding lecture gave the sun-spot theory which is the subject of this paper.

1. *Dynamics of circulation.*—The internal dynamics of a perfect fluid depends upon two fields, that of *pressure* and that of *mass*. In the field of pressure the *pressure gradient* points in the direction of decreasing pressure and represents the force per unit volume originating from pressure. In the field of mass the *inertia gradient* points in the direction of decreasing inertia, i.e., of decreasing density or increasing specific volume. If the two vectors do not coincide, the inertia will be asymmetrically distributed relative to the force due to pressure. This will give rise to rotation about an axis normal to the two vectors, tending to bring the densest masses in the rear. The rotation will thus be directed from the inertia gradient toward the pressure gradient.

To give the law quantitative form we may represent the field of pressure by isobaric surfaces drawn for unit differences of pressure, and the field of mass by surfaces of equal specific volume (isosteric surfaces) drawn for unit differences of this quantity. These surfaces divide space into a set of unit tubes, characterizing the asymmetry which the field of mass presents relative to the field of pressure. The positive direction of rotation around these tubes is by definition that mentioned, i.e., from the inertia gradient to the pressure gradient. Defining with Lord Kelvin the circulation along a closed curve as the line integral of the tangential component of velocity,

$$C = \int v_t ds \,, \tag{1}$$

and calling N the number of unit tubes comprised by the curve, we have

$$\frac{dC}{dt} = N \,, \tag{2}$$

i.e., *any closed fluid curve has at any time a rate of increase of circulation in the positive direction round the unit tubes which is numerically equal to the number of these tubes included within the curve.*[1]

Classical hydrodynamics restricts its considerations to "baro tropic" conditions, according to which the field of pressure entirely

[1] Cf. V. Bjerknes, "Ueber die Bildung von Cirkulationsbewegungen und Wirbe'n in reibungslosen Flüssigkeiten," *Videnskabsselskabets Skrifter*, Christiania, 1898.

governs the field of mass. The inertia gradient then coincides with the pressure gradient, no unit tubes exist, $N=0$, and (2) reduces to

$$C=\text{const.} \tag{3}$$

Thus, the circulation of any closed fluid is invariable. This is the Helmholtz-Kelvin principle of the conservation of circulations and vortices. All circulations in fluid or gaseous masses, such as those surrounding or constituting the celestial bodies, must be according to law (2), and may be calculated from it if we can determine the isosteric surfaces, or the equivalent isothermal surfaces, *as something different from the isobaric surfaces*. When, vice versa, variations of circulation are known, we may derive the corresponding numbers of unit tubes, and thus derive relative distributions of mass and pressure. Only one example, which is instinctively understood, will be given here:

Let a horizontal curved current be given, which for simplicity is assumed to be steady or approximately steady. If the current has an upward-increasing intensity, the upper masses will have an excess of centrifugal force. This produces a centrifugal pumping effect, which lifts heavy masses (adiabatically cooled gas) on the inner concave side of the current. If, vice versa, the current has an upward-decreasing intensity, the lower masses will have an excess of centrifugal force. This causes a centrifugal sucking effect, which brings light masses (adiabatically heated gas) down on the inner concave side of the current. A vortex with an upward-increasing velocity will, therefore, have a core of heavy masses or cold gas; a vortex with upward-decreasing velocity, a core of light masses or of hot gas.[1]

The theorem of circulation (2) gives completely the dynamics of these pumping and sucking effects, and leads to a convenient formula characterizing the equilibrium conditions ultimately reached. In deriving such a formula, we may use the gas equation, and thus introduce the absolute temperature ϑ instead of the specific volume. Let dv be the increase in velocity for an increase in height dz, g the acceleration of gravity, $d\vartheta$ the increase in temperature for an increase in

[1] Cf. V. Bjerknes, "On the Dynamics of the Circular Vortex with Applications to Atmospheric Vortex and Wave Motion," *Geofysiske Publikationer*, Christiania, 1921.

length ds measured along a curve s which is contained in the isobaric surface, runs normal to the lines of flow, and is counted positive outward, i.e., from the concave to the convex side of the current. Finally, on account of the curvature of the current, the moving particles have a certain angular velocity Ω around an axis which forms the angle ϕ with the isobaric surface. Then Ω represents the component of angular velocity round the normal to the isobaric surface. We then have the formula

$$\frac{1}{\vartheta}\frac{d\vartheta}{ds}=\frac{2\Omega \sin \phi}{g}\frac{dv}{dz} \tag{4}$$

which may be used as long as the vertical accelerations are zero or moderate and the state of motion steady or nearly steady.

The principle of pumping or sucking effects of curved currents takes a slightly different form when, from absolute motion, we pass to the motion relative to a rotating body like the earth or the sun. A current which is straight, relative to the rotating rigid body, will be curved from the viewpoint of absolute motion. Instead of referring to the concave and convex side of the curved current, we refer to the left and right side of the apparently straight current, and find the following rule:

In the northern hemisphere of the earth or the sun a current will carry cold masses on its left side, if it has an upward-increasing intensity, and on its right side, if it has upward-decreasing intensity.[1]

As an example, we may mention that the general atmospheric motion of the northern hemisphere of the earth is an eastward drift with upward-increasing intensity, which carries the cold polar air masses on its left side. Formula (4) applies directly to these relative motions when ϕ is the latitude and Ω the angular velocity of the earth or the sun.

A more complex formula can be derived to cover the case of a current which appears curved in its relative motion. But the simple formula (4) will be sufficient, if in each case we interpret Ω as the greater of the two angular velocities: that of the vortex, and that of the solid rotating body, the sun or earth.

[1] Cf. T. W. Sandström, "Ueber die Beziehung zwischen Temperatur und Luftbewegung in der Atmosphäre unter stationären Verhältnissen," *Öfversigt af K. Vetenskaps-akademiens Förhandlinger*, Stockholm, 1901; *Meteorologische Zeitschrift*, **19**, 161, 1902.

2. *Thermodynamics of circulation.*—To illustrate, from a thermodynamical viewpoint, the conditions for producing circulation, consider a closed tube, for simplicity, of uniform section, filled with fluid or gas in adiabatic, i.e., indifferent, equilibrium. The slightest heating of a single element E will disturb barotropic conditions, and the whole fluid mass will come into a circulating motion which will not stop until this element has reached the highest point of the tube.

To maintain circulation systematically against a resisting force such as friction, we can, in one branch of the tube, at a certain height, place a source of heat (Fig. 1, A) which the fluid cannot pass without

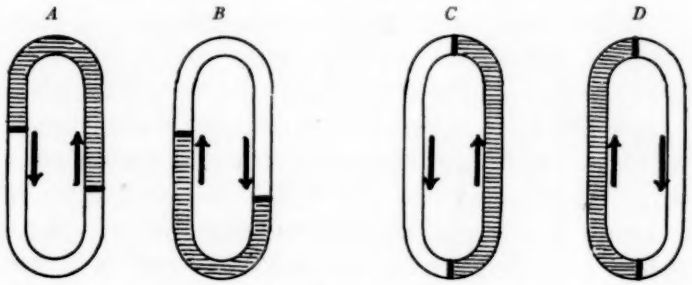

FIG. 1

assuming a definite increase of temperature and, at a certain other height in the other branch, a source of cold which the heated fluid cannot pass without a corresponding drop in temperature. The entire fluid mass will then be kept in circulation by a force which is equal to the difference in weight between two fluid columns of different temperatures but of the same height, the height being equal to the difference in elevation of the two sources. The circulation considered is upward through the branch containing the source of the heat, and down the branch containing the source of the cold. The tube becomes filled with heated fluid above the two sources, and with cooled fluid below.

It is an important peculiarity of the engine that it may run in the opposite direction. For this it suffices that, by an impulse, we produce a certain initial retrograde circulation (Fig. 1, B). The fluid passing down through the source of heat will then be heated, while that moving up through the source of cold will be cooled. The two

columns of different temperatures have thus interchanged places, and during the motion the part of the tube below the two sources will contain warm fluid, and that above them, cold fluid. The efficiency of the engine is exactly the same for direct and for inverse motion. The engine is self-starting for direct motion, while an initial impulse is required to start the retrograde circulation.

The efficiency is greatest when the source of heat is at the lowest point and the source of cold at the highest point of the tube (Fig. 1, *C* and *D*). In this case the difference between direct and retrograde motion disappears. The engine then ceases to be self-starting, but an infinitely small impulse will start it in either direction; and, when started, it then works with higher efficiency than for any other arrangement of the sources.

When the source of cold is at a lower level than the source of heat, the efficiency is negative, i.e., the engine will transfer more heat to the source of cold than it receives from the source of heat. Instead of transforming heat into kinetic energy, it will transform kinetic energy into heat as long as there exists a circulating motion from which kinetic energy can be derived.

3. *Stratified circulation.*—In the preceding example the fluid is supposed, initially, to be in barotropic equilibrium. Whatever the form of the vessel, the slightest heating of a single element then brings the whole mass of fluid into circulation, which continues until the heated element has reached the highest position possible. Similarly, a cooled element cannot stop before it reaches the bottom.

It is quite different if the fluid is intrinsically heterogeneous, with heavier masses below and lighter masses above, corresponding to stable internal equilibrium. A heated element will then ascend only to the level of fluid having its own density. In common experiments with small fluid masses a heated element, as a rule, soon reaches the surface and the heterogeneities are rapidly smoothed out. But when the volume is large in comparison with the differences of temperature, we meet with the very characteristic phenomenon of *stratified circulation*. Very instructive is the following experiment due to Alf Sinding-Larsen.[1]

[1] *Annalen der Physik*, **9**, 1190, 1902.

A test tube containing a heterogeneous salt solution is put in a glass of hot water. The salt solution then divides into a series of superimposed strata, each having its independent circulation (Fig. 2, *A*). If after heating, the tube is put in cold water, the opposite circulation occurs within the strata (Fig. 2, *B*). Circulations produced in this way are symmetrical with respect to the axis of the tube. Circulations across the tube may be obtained by heating it on one side and cooling it on the other. In these experiments the circulation evidently tends to produce homogeneity within each stratum, with the result that the heterogeneities are concentrated in narrow transitional layers separating the homogeneous strata. If the physical and chemical conditions of the fluid permit the formation of a single homogeneous stratum, such a stratum will be the ultimate result, the existence of several strata being merely a transitional state; but if the strata pre-exist through physical or chemical reasons as with oil and water for instance, they will resist this tendency to form a single homogeneous mass.[1]

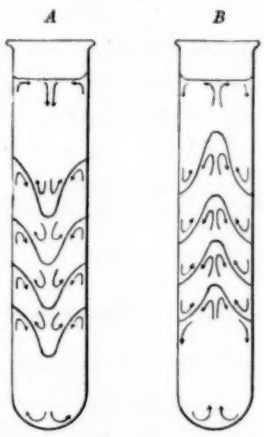

FIG. 2

In the preceding experiment heating produced and friction counteracted the circulation. But under favorable circumstances friction may also produce stratified circulation. Experiments and

[1] It does not seem necessary to assume with Sinding-Larsen that in his experiments the strata are pre-existent because of difference in molecular structure, and merely become visible in consequence of circulation. The limited ascending power of the heated masses in the heterogeneous fluid system seems sufficient to explain the formation of the strata and to be in full accordance with his results: namely, that the strata are the more striking the greater the difference between the specific weights of the solvent and the concentrated solution; that the number of strata seems to depend partly upon the differences of concentration in the solution; that the depth of the strata increases in general with decreasing concentration; that the bounding surfaces of the strata move upward, in general, as a consequence of the steadily increasing depth of the individual strata, from which it follows that the number of strata decreases.

My attention has only recently been directed to a paper by Mendenhall and Mason, "The Stratified Subsidence of Fine Particles," *Proceedings of the National Academy of Sciences*, **9**, 199, 1923, in which stratified circulations of thermal origin are examined.

observations on the phenomenon have been made especially by J. W. Sandström.[1]

Consider a series of superimposed strata, each stratum for simplicity being a homogeneous salt solution, with a sudden increase in salinity as we pass from one stratum to that next underlying (Fig. 3). A wind on the free surface of the top stratum will set this stratum in circulation. By friction the next underlying stratum then comes into opposite circulation. This in turn acts upon the next lower stratum, and so on, with the result that all strata circulate like toothed wheels, but with decreasing intensity as we proceed from

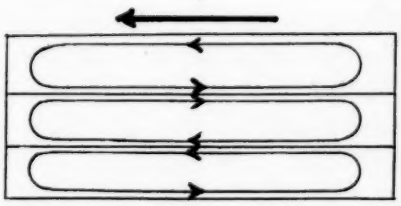

FIG. 3

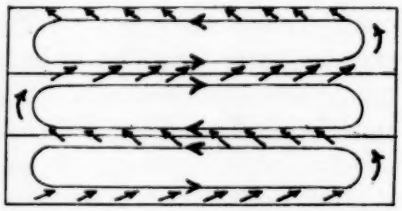

FIG. 4

stratum to stratum downward. Sandström also describes phenomena of this type produced by the wind on stratified sea water along a beach.

We have supposed the homogeneous superimposed strata to be pre-existent. But if the solution has initially a continuously varying concentration, the wind will evidently tend to produce an upper homogeneous circulating layer, which will then tend to produce a second homogeneous layer immediately below, and so on. It would be interesting to examine experimentally how far this process can proceed.

In the experiments of Sinding-Larsen, friction counteracts the circulation produced by heating. But evidently conditions may arise such that heat and friction co-operate in producing and maintaining stratified circulation. Figure 4 illustrates a possible motion where this is the case. The fluid mass is supposed to be heated from below and cooled at its free surface. Initially conditions are unstable, as

[1] "Windströme im Gullmarfjord," *Svenske Hydrografiske-biologiske Kommissionens Skrifter*, II, Göteborg, 1905.

in the tubes of Figure 1, *C* or *D*. But an initial impulse may be used to determine the direction of circulation in the lowest layer. Friction will then give an opposite circulation to the next higher layer, and so on. But friction will not be the only cause maintaining the circulation of the higher layers. Each of them will be heated from below, and take part in the convection transport of heat. The small arrows in Figure 4 indicate the flow of heat.

It would also be interesting to test experimentally the conditions for realization and stability of this kind of stratified circulation, both when the strata pre-exist and when formed as a result of the circulation. The initial impulse determining the direction of the circulation could be avoided by slightly inclining the heated bottom, and an inclined rigid surface which is cooled could be substituted for the upper horizontal free surface.

II. EMPIRICAL DATA CONCERNING THE SUN

4. The sun rotates on its axis in about twenty-five days. The rotation is not that of a rigid body, the angular velocity being considerably greater at the equator than in higher latitudes. The highest layers in the sun's atmosphere seem to follow closely the motion at the equator. Thus, in passing from the photosphere upward in the atmosphere of the sun, we approach, in all extra-tropical latitudes, layers of increasing velocity.

5. Sun-spots seem to be depressions in the photosphere. They occur in lower latitudes, and seldom appear as much as 45° from the equator.

6. The sun-spots of each cycle always appear within two relatively narrow zones 10° or 15°, or at most, 20° wide, symmetrically situated in the two hemispheres. These zones are most distant from the equator at the beginning of a cycle, and during the eleven years of the cycle gradually approach, so that toward the end they may almost reach the equator; but they never cross it. Before the spots of one cycle have disappeared, those of the next have made their appearance in the higher latitudes and begun their motion toward the equator.

7. In the majority of cases sun-spots appear in pairs forming binary systems. The axis of a binary system is, on the average, near-

ly parallel to the equator. In general, the closer the parallelism, the nearer the system is to the equator.

8. Sun-spots always show magnetic polarity, the members of a binary system having opposite polarities. A single spot may consist of two halves of opposite polarity, and thus, from the magnetic point of view, be a binary system. At the time of their appearance spot groups very frequently are bipolar. They usually end as single spots.

9. The preceding spots of binaries, almost without exception, have during the whole cycle a definite polarity, characteristic of that cycle. The polarities of preceding spots are opposite in the two hemispheres, and the whole system of polarities reverses with the beginning of a new cycle.

10. Within a binary system a certain asymmetry generally prevails. The preceding spot, as a rule, is the larger and has the longer life; the companion is smaller, often consists of a number of smaller spots, and may disappear long before the preceding spot.

11. Hydrogen spectroheliograms in the region of sun-spots often show spiral structure. The curvature is usually opposite in the two hemispheres, but does not change from cycle to cycle. If the spiral structure represents lines of flow, the motion must be both radial and circular. Since it has been obtained only by the use of hydrogen lines, it is believed to belong to the high levels of the solar atmosphere and to be merely a secondary effect of what is going on deeper down, in or below the photosphere. Occasionally hydrogen clouds have been drawn into a spot, with velocities up to 170 km/sec. Motion outward from the spot in the lower layers of the sun's atmosphere has also been observed.

The line structures surrounding bipolar systems seem sometimes to suggest an outflow from one spot and a corresponding inflow into the other; but the structures are generally complicated and not easy to interpret.

III. GENERAL SUPPOSITIONS CONCERNING THE SUN

As a basis for an attempt to interpret the observed phenomena, the following suppositions will be made:

12. As the sun consists of gases of different specific weights, it will be near a state of *stable* internal equilibrium; i.e., the equilibri-

um is not of the adiabatic or indifferent type which would exist if density (and temperature) varied downward as a function merely of pressure. The difference may be illustrated thus: In the case of adiabatic equilibrium a mass which has been cooled at the sun's surface would sink to the sun's center, but in the case of stable internal equilibrium it would sink only to a certain equilibrium level from which it would ascend again when heated to the temperature of this layer.

Conditions in the sun will, therefore, be favorable for the formation of stratified circulation, and the conditions for this kind of circulation will be still more favorable if relatively homogeneous strata pre-exist, such that each stratum consists mainly of one chemical constituent or of constituents having approximately the same specific weight, but separated from the higher and lower strata by transitional layers in which chemical constitution and corresponding physical properties are subject to rapid variations.

13. The origin of all that happens on the sun is in the solar radiation. This may be taken to include the expulsion of electrically charged particles, which may be of importance for the electric conditions of the sun. The luminous and thermal radiation, on the other hand, will be considered responsible for the internal dynamics of the sun. It cools the upper layers of the photosphere and thereby disturbs the equilibrium conditions.

14. The heat thus lost by the photosphere is restored from the deeper layers by internal radiation, convection, and conduction. Internal radiation is believed to be the dominant factor, especially in the deep layers of very high temperature. But a sufficient percentage is left for setting up powerful convectional currents. The importance of convection increases near the surface, where, on account of the lower temperature, the internal radiation becomes less dominant.

15. The rotation of the sun acts as an organizing factor, tending to produce a general zonal symmetry around the axis of rotation.

16. The sun-spots are a kind of vortex. No hypothesis is made concerning the origin of their magnetic fields, but it is supposed that the fields are connected with the vortical motion in such a way that the magnetic polarity is reversed by a reversal of the vortex motion.

IV. SUGGESTED INTERPRETATION OF THE SOLAR PHENOMENA

17. *Vortex theory of sun-spots.*—The well-known comparison of sun-spots with tropical cyclones involves the assumption that dynamically a sun-spot is a vortex. If this is correct, the comparison of sun-spots with the greatest vortices known on the earth, the tropical cyclones, will be useful in many respects, and, first, may give us a mental picture of the dimensions. The diameter of a small tropical cyclone is 120–130 kilometers. Let us now enlarge both the earth and the cyclone a hundred times. The earth will then have approximately the dimensions of the sun, the cyclone will be of the dimensions of the equatorial plane of the earth, and thus represent the area of a good-sized sun-spot. We know nothing of the height or depth of the sun-spot vortex; but taking again a hundred times the height of a tropical cyclone, which is between 5 and 10 km, we arrive at 500 to 1000 km. The body of a fair-sized sun-spot vortex would then be represented by the section contained between the equatorial plane of the earth and a parallel plane between 4° and 9° latitude.

A first fundamental question to be examined is whether a vortex theory can account for the visibility of sun-spots, i.e., of their relative darkness or low temperature. Formula (4) may be used for discussing this question. It gives the temperatures in what we may call a balanced vortex, in which the cooled heavy masses are just carried by the centrifugal pumping effect, and no ascending motion is present. Sun-spot vortices will, as a matter of course, not have this balanced character. The lifted cooled masses are subject to continuous radiation from below, and the pumping must go on continuously to lift the cooled masses at a sufficient rate to keep them sufficiently cool in spite of the radiation. Nevertheless, the formula for the balanced vortex will give a useful first approximation. To bring it into a convenient form for our special purpose, we remark that $ds \sin \phi$ is the element dr of a radius vector from the axis of the vortex. We may thus write:

$$\frac{d\vartheta}{\vartheta} = \frac{2\Omega}{g} \frac{dv}{dz} dr$$

Then, for the sake of argument, we shall make a simple assumption concerning the rate at which the velocity v increases with the height

z in the vortex. For simplicity let the velocity be zero at a certain depth H below the surface of the photosphere and increase upward according to a linear law until it reaches the value v or Ωr at the surface of the photosphere. We may then substitute

$$\frac{dv}{dz}=\frac{v}{H}=\frac{\Omega r}{H}$$

and thus obtain

$$\frac{d\vartheta}{\vartheta}=\frac{2\Omega^2 r}{Hg}\,dr\,.$$

But the ratio of the centrifugal force $\Omega^2 r$ to the acceleration of gravity g is the tangent of the angle of inclination of an element ds of the

FIG. 5

photosphere, and this multiplied by the horizontal line element dr gives the vertical projection dz of a line element ds contained in the inclined surface of the photosphere. Integrating this equation along the surface of the photosphere from a great distance to the center of the vortex, we obtain a relation between the drop in temperature $\Delta\vartheta$ and the corresponding dip D below the normal level of the photospheric surface (Fig. 5). For simplicity we may assume the thickness H of the vortex to be constant. Considering at the same time the drop in temperature $\Delta\vartheta$ to be small as compared with the temperature itself, we find by the integration the simple formula

$$\Delta\vartheta=2\vartheta\,\frac{D}{H}\,. \tag{5}$$

For large values of the drop in temperature we must use the result of the exact integration, viz.,

$$\log_e\frac{\vartheta_0}{\vartheta}=\frac{2D}{H}\,. \tag{6}$$

The approximate formula (5) represents the following exceedingly simple law: The drop in temperature is proportional to the dip in the surface of the photosphere. The dynamics of the phenomenon is obvious: The dip is due to the excess weight of the relatively cool gases forming the core of the vortex; these are prevented from leaving the surface by the centrifugal pumping effect arising from the excess of circulatory motion in the higher strata. On the other hand, for the same value of the dip, the drop in temperature is inversely proportional to the thickness H of the vortex; the thinner the cooled stratum, the more must it be cooled to give the required excess of weight.

Introducing $\vartheta = 6000^\circ$ as the surface temperature of the sun, we find

$$\Delta\vartheta = 12{,}000 \frac{D}{H}$$

or, in the exact logarithmic form,

$$\log_e \vartheta_0 - \log_e \vartheta = 0.8686 \frac{D}{H}.$$

These formulae give numerically

D/H	0.001	0.01	0.1	0.2	0.3	0.4	0.5
$\Delta\vartheta$	12°	120°	1100°	2000°	2700°	3300°	3800°

Thus a dip in the photosphere amounting to only one-tenth of the thickness of the stratum taking part in the vortex motion will be sufficient to produce a drop in temperature of 1100°, while a greater dip will produce an even greater decrease in temperature. Now let the diameter of the vortex be ten times its thickness. The average inclination required to produce the dip of 1/10 would then be 1/100. The velocities required to produce this inclination at different distances r from the axis of rotation are

$r =$	10	100	1000	10,000	100,000 km
$v =$	0.2	0.6	1.7	5.5	17.5 km/sec.

For the conditions on the sun these velocities seem very moderate. For a fourfold inclination, which gives a temperature drop of

3300°, the velocities would have only to be doubled. They thus remain small compared, for instance, with the velocity of 170 km/sec. found for a hydrogen cloud in the sun's atmosphere, not to speak of the amazing velocities observed in the prominences. There will be no difficulty therefore in accounting for the somewhat greater velocities which would be required to give the same temperature depressions in the unbalanced vortices. The simple case of the balanced vortex will, however, give the easiest available first approximation. Detailed examples of such sun-spot vortices, with full representation of the fields of velocity, pressure, mass, and temperature, may easily be worked out by use of the formulae given in my paper on the circular vortex, quoted above.

An important question is also the depth from which the vapors must be pumped up in order to give the required drop in temperature, for instance, 1100°. The adiabatic decrease of temperature in the earth's atmosphere is 1° C. per hundred meters of height. On the sun, with gravity 27 times stronger, the decrease would be 27° per hundred meters for an atmosphere like that of the earth, and we may assume for the solar gases a value differing not greatly from this. When these gases are lifted 10,000 meters or 10 km, they should cool down 2700°. How much this would reduce their temperature below that at the new level depends upon the solar temperature gradient. If, as in the earth's atmosphere, this is half the adiabatic gradient, the result would be 1300°. For smaller differences between the actual and the adiabatic rate, the required vertical displacement might be some multiple of 10 km. But great vertical displacements will not come into the question.

The vortex theory thus gives a straightforward explanation of the darkness of sun-spots; no great effort of any kind is required to explain their low temperatures. As a matter of fact, the trouble is, not to explain why sun-spots are seen, but rather why many more of them do not become visible and then disappear on a body where conditions for the formation and annihilation of vortices are as favorable as on the sun. This raises the question of the long life of a sun-spot, which may extend to months or to even more than a year. Under the intensive radiation from below, the cooled masses forming at any moment the core of the vortex cannot long retain

their low temperature. But as they become heated they will be replaced by new masses pumped from below and float out radially over the surface of the photosphere. This process will continue as long as sufficient energy is at hand for the pumping operation. During the outward motion the circulation around the center of the spot will rapidly decrease, and the radial component will prevail. We have emphasized this radial outflow along the surface of the photosphere as a typical characteristic of sun-spots.

The radial motion outward at the bottom of the sun's atmosphere, as remarked by Hale, must give rise to a sink and a corresponding radial inflow in the upper strata. Because of the rotation

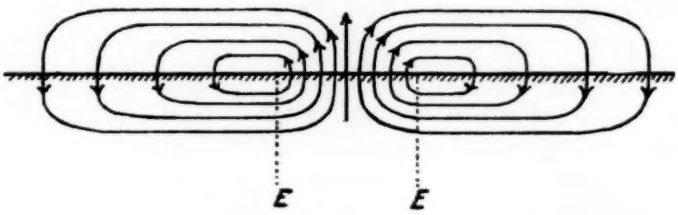

FIG. 6

of the sun the inflow will assume a spiral character, which may explain the spiral structure sometimes appearing in the high layers. Since this is determined by the direction of the sun's rotation and not by that of the photospheric vortex below, these spiral structures would have no reason to change when the magnetic polarities change at the beginning of a new sun-spot cycle.

For further discussion of the structure of sun-spot vortices, we shall make use of the vortex lines. Each element of a vortex line is an axis of local rotation, the direction of which is given by the positive screw rule. The vortex line, by geometric necessity, can never stop; it must run back into itself. This is also the fundamental property of the lines of magnetic induction. This common property facilitates the parallel discussion of the vortical and magnetic properties of sun-spots. In the first rough approximation, the vortex lines and the lines of magnetic induction may be identified. Where these lines are strongly concentrated we thus have intensive vorticity and

strong magnetic field; where they are dispersed, we have weak vorticity and weak magnetic field.

The simplest hypothesis which can be made is that each sun-spot is an independent vortex. Figure 6 gives the vortex-line representation of such a vortex. The lines come up in great numbers through the photosphere in the central part of the vortex, spread out in the solar atmosphere, and return through the photosphere to the central vortex region. We have no idea as to the heights to which the vortex lines ascend in the solar atmosphere. Most likely they are concentrated near the surface of the photosphere, thus representing here

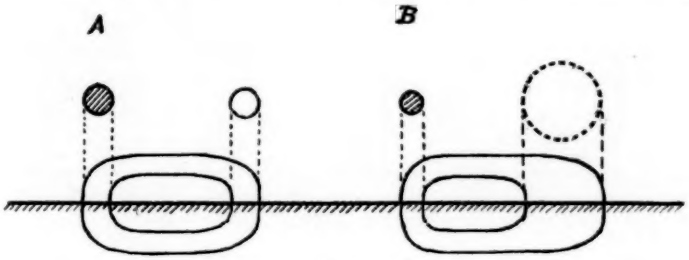

FIG. 7

an intensive sliding motion. As the visibility of the sun-spot demands a rapid decrease of velocity downward, the vortex lines will not reach great depths below the photosphere. At the surface itself, between the points E and E', we have a limited area of intensive positive vorticity and strong magnetic field; outside we have weak negative vorticity and a weak magnetic field, opposite to that in the internal area. We do not know the correspondence between the direction of rotation and the sign of the magnetic field, and a reliable determination thereof would be of fundamental importance for the theory of sun-spots, and perhaps also for that of magnetism.

If each sun-spot were an independent vortex, a binary system would appear only by accident, two vortices of opposite rotation happening to be near each other. To explain the binaries Hale has therefore suggested a coupled system forming a vortex ring, the two spots of the binary being the intersection of this ring with the photosphere. One half of the ring then belongs to the atmosphere, while the other half, dynamically the more important, forms a sub-photospheric connection between the spots (Fig. 7, A). Nothing prevents

us from assuming that all sun-spots belong to binary systems of this type, for a spot which appears single may have an invisible companion, the vortex ring cutting the surface so diffusely that nothing is observed (Fig. 7, *B*).

The vortex-ring theory gives a simple explanation of the tendency of sun-spots to appear as binaries; but it fails to give any natural explanation of the characteristic collective properties of spots, which seem to point strongly toward a more complete coupling, not only between the spots of the same binary but also between the different binaries of the same cycle. As a matter of fact, a com-

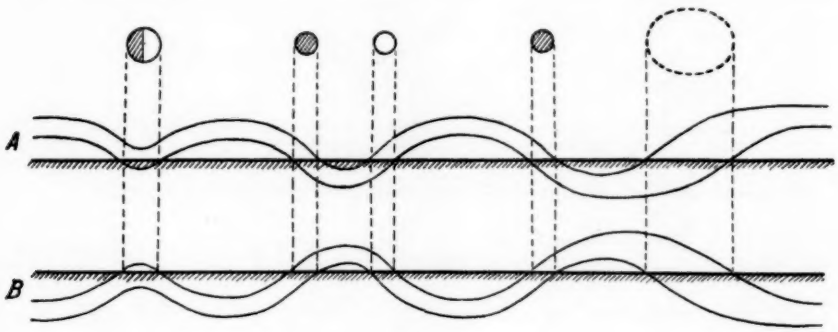

FIG. 8

plete vortical connection between all spots of the same cycle can also be imagined. Thus all the spots may belong to a single zonal vortex, which surrounds the sun approximately as a parallel and gives rise to a spot, wherever it passes from the photosphere to the atmosphere or vice versa. The assumption may be made in two different forms: The zonal vortex may belong mainly to the atmosphere and occasionally dip below the photosphere from spot to spot of a binary (Fig. 8, *A*); or, vice versa, it may be mainly subphotospheric, and occasionally ascend into the atmosphere from member to member of a binary system (Fig. 8, *B*). Leaving until later the choice between these alternatives, we shall first derive the consequences which are common to both forms of the zonal vortex theory.

While conserving the advantages of the vortex-ring theory, the zonal vortex gives a simple picture of the most striking collective

properties of sun-spots, namely, that spots of the same cycle appear within a relatively narrow zone of latitude; that spots appear generally in binary systems; that the axis of a binary tends to be parallel to the equator; that binaries of the same cycle, almost without exception, show the same succession of polarities; that groups at their first appearance are very frequently bipolar; that a single spot is generally the last phase, one member of the binary system becoming diffuse and invisible before the other; that a companion to a single spot appears in the rear, if the original spot has the leading polarity for the cycle, and ahead if the opposite state of affairs prevails.

In addition, the theory reduces the progression of sun-spot phenomena from higher to lower latitudes to a motion of the single zonal vortex, while the reversed succession of polarities from cycle to cycle is explained as a reversal of the rotation of the zonal vortex.

The various advantages which the zonal-vortex theory of sun-spots thus seems to possess makes it desirable to find ways of testing its possibility. The simplest preliminary test would be to plot from day to day the probable course of the zonal vortex, in order to see if this gives an adequate representation of the distribution and propagation of sun-spots. In doing this it is recommended that areas of different polarity be marked with different colors, say red and blue. The simplest curve is then drawn which connects these areas in the regular succession red, blue, red, blue, etc. When necessary, a hypothetical invisible spot is to be inserted where the regular succession of polarities requires it. Remarking that this is permissible, and, further, that a vortex tube is infinitely flexible and may branch out and join again, we see that it will always be possible to draw the required curve. The difficulty is that its course is not uniquely determined. The aim should, therefore, be to draw it with the smallest possible number of assumed invisible spots and with as few branches and sinuosities as possible. Figure 9 indicates a possible course for such a curve, illustrating both a division into two branches and an S-shaped return of the curve used to explain the case of reversed polarity.

If it should prove possible to draw these curves so that they do present the distribution and progression of sun-spots, and if the

survey of the sun's surface, magnetically and optically, along the course of the curves facilitates the discovery of invisible sun-spots, etc., the zonal vortex theory would at least be useful as a working hypothesis.

All the collective properties of sun-spots enumerated follow from the zonal-vortex theory in both its forms; but there is a decided reason in favor of the assumption that the zonal vortex is mainly sub-photospheric. Just below the radiating stratum of the photosphere we shall find the strongest tendency to form convective circulations of thermal origin. A sub-photospheric vortex, along its whole length, will bring hot gases to the surface and carry cooled

FIG. 9

gases down again. Throughout its entire extent it is a thermodynamically driven engine of the type Figure 1, *C* or *D*, and has the important property of running in both directions equally well, which provides for the two directions of rotation required for reversing the succession of polarities from cycle to cycle. That such a vortex does not take an irregular form, but tends to lie parallel to the equator, follows from the rotation of the sun, which favors symmetrical arrangements relative to the axis of rotation.

Finally, it may be asked why such a sub-photospheric vortex should occasionally ascend to the surface. To answer this question we must recall the attraction or repulsion existing between parallel vortices and the attraction of a vortex toward a rigid plane. The surface of the photosphere, on account of its stability, will act as a practically rigid plane, which in general will attract the vortex. The force will disappear in one case only, namely, when the vortex yields completely to the current which tends to carry it along parallel to the surface. But such a complete yielding is excluded in the case of a vortex maintained convectively and receiving its energy from

hot strata which never have the exact motion of the upper current. The attracting force will, therefore, always exist; and, if the vortex has an upward sinuosity, this sinuosity will come under increased attraction, as the force varies inversely as the distance from the plane. Ultimately the sinuosity may thus reach and cut the surface and form the two separate horizontal vortices of a binary, and even after this the continued attraction will tend to bring more of the vortex up and thus separate the two spots from each other.

There is another reason also, and perhaps more important, for the tendency of the sub-photospheric vortex to rise and cut the photosphere. The regular state below the surface of the sun is one of stable stratification. Circulation can take only the stratified form, with limited vertical excursions, whereas no special forces limit the horizontal excursions. The sub-photospheric vortex must, therefore, be very flat, the circulating curves having very flat elliptic forms, with horizontal major axis and vertical minor axis. This is a strained type of circulation, guided by a certain balance of forces. It may, therefore, have a strong tendency, when an opportunity offers, to turn over in the horizontal plane. Here a circulation, once started, goes on undisturbed, as a perfectly stable motion. This tendency to realize the most stable motion may, therefore, be the main reason why the vortex rises and cuts the photosphere, and why afterward new lengths of the photospheric vortex are drawn up to the already-formed horizontal sun-spot vortex, thus giving it a continuous supply of sub-photospheric energy.

When the vortex tube is thus bent up, the strained elliptic circulation in the vertical plane changes automatically into a circular circulation in the horizontal plane (Fig. 10). For the same area of cross-section this gives much shorter circulating curves, and therefore an intensified circulation. This intensified circulation again gives the pumping effect which lifts up gases from below and makes the spot visible by its lowered temperature. The kinetic energy of this localized vorticity is by and by absorbed in the work of pumping. But new energy becomes available according as new lengths of the sub-photospheric vortex are raised to the surface. In this way the sub-photospheric vortex may be drawn upon for energy for any length of time, and secure to the sun-spot any length of life.

18. *General solar circulation.*—Our view of sun-spots as a link in the internal circulation of the sun brings up the question of the general character of this circulation.

As in all likelihood strata of different density pre-exist in the interior of the sun, it seems very probable, if not necessary, that the circulation should be of the stratified type. In addition, the circulation itself will tend to develop the stratification further. The main question will be the vertical and horizontal extent, and the mutual arrangement of the individual circulating strata. Here the rotation of the sun enters as a determining factor. Without this rotation there would be no organizing principle. The stratification would have an irregular and turbulent character; but with rotation there

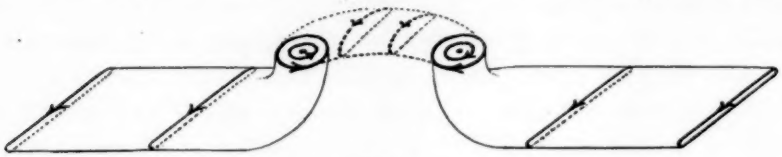

FIG. 10

must enter a tendency toward symmetry relative to the axis of the sun. How far the organizing power of the solar rotation may extend can be only vaguely estimated.

An example of the simplest possible scheme of stratified circulation is given in Figure 11. Here the highest stratum is supposed to have a circulation from the poles to the equator at the surface, and from the equator back to the poles below. The next lower stratum should then circulate from equator to pole in its upper part, and from the pole to the equator below, and so on. One may ask why the highest layer should not rather circulate as in the case of the earth's atmosphere, namely, from equator to poles in the higher levels and from poles to equator below. The answer is that the circulation of the earth's atmosphere is self-starting in the direction observed because the main source of heat is at the ground level at the equator. On the sun, to the contrary, it is difficult to see why the circulation should start in either direction in preference to the other, since we have initially a symmetry corresponding to that of Figure 1, *C* and *D*. But *if* the circulation is started in a given direction, it should then persist in that direction.

The motive power of the circulation in the upper stratum is the cooling of the photosphere by radiation, and the corresponding heating lower down by radiation from within and by contact with the next lower stratum. Since the power of radiation decreases outward as the temperature decreases, the lowest part of the stratum has better access to the radiated heat than the upper part. This will favor heat transport by convection. The next lower

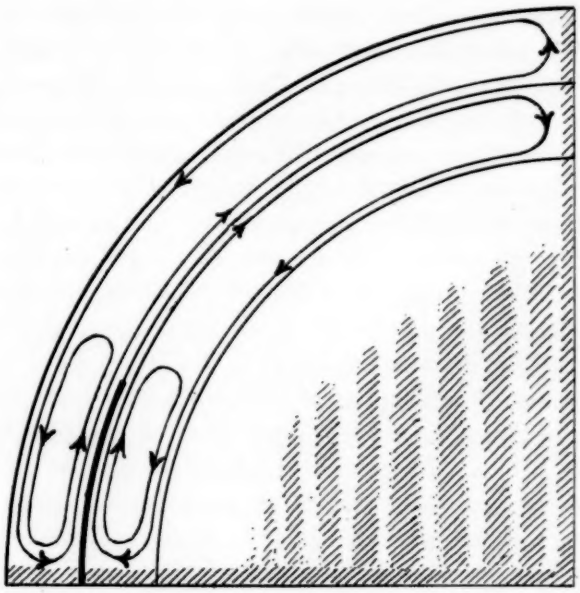

FIG. 11

stratum then comes into circulation by frictional effect, assisted by the cooling of its upper part by contact with the upper stratum and the heating of its lower part by the hot gases below.

If such a circulation exists, the surface of the sun as seen from the earth will not rotate as a rigid body. We cannot define any definite angular velocity of the entire solar surface, but only a certain zonal distribution of angular velocity, the character of which we shall now examine.

Evidently those parts of the sun which participate least in the circulation will move most nearly as a rigid body. These will be the

great internal masses on both sides of the equatorial plane and surrounding the axis of rotation. The corresponding areas are shaded in Figure 11. Then for the masses participating in the circulation we must use the general law, so well known from the earth's atmosphere, that masses moving from pole to equator lag behind, while masses moving from the equator to the poles advance relative to the rigid, or apparently rigid, core. This always occurs to such an extent that the main motion becomes pratically a pure EW motion, accompanied only by a slow displacement of the masses in the NS direction. The upper layer of the highest circulating stratum should, therefore, in all latitudes appear retarded relative to the equatorial masses. Thus the internal circulations of the sun seem to open the possibility of explaining the so-called *equatorial acceleration*, which, from this viewpoint, should rather be called a *retardation in the higher latitudes*. A difficulty for this theory may be met in the fact that, as far as we can see, a retardation relative to the pole is to be expected; in other words, a polar acceleration should appear near the pole, where it will be difficult, however, to obtain good determinations of the angular velocity.

The amount of retardation of the upper masses will be limited mainly by friction against and mixing with the lower masses of the layer, which have a corresponding acceleration because of their motion from the equator to the poles. On the other hand, the fact that the observed retardation in the higher latitudes is so moderate shows that the motion from the poles to the equator must be very gradual, thus leaving time for an effective smoothing out of the difference in velocity between the retarded upper part and the accelerated lower part of the stratum. Observations indicate that the retarded masses in moderately high latitudes have had their angular velocities around the sun's axis reduced by about 1/20. This would represent an east wind relative to the quasi-rigid body of the sun which would traverse the solar circumference in about 20 solar days or in something more than a terrestrial year. The corresponding velocity would be about 130 meters per second. The motion from the poles to the equator should, therefore, require several years, and the corresponding north wind would be very weak.

The sub-photospheric zonal vortex from which sun-spots develop

must belong to the lower, slow-moving layers of this east-current stratum. Then, when the vortex tube rises, cuts the photosphere, and forms a binary spot, the two separated parts of the tube must be affected differently by the stronger east wind near the surface. Attention is directed to this as a possible origin for the asymmetrical properties of binaries, although no attempt to consider details is made at present.

During their slow motion from the poles to the equator, the masses forming the surface of the photosphere must be gradually cooled. In consequence the sun should have its highest temperature at the poles and its lowest at the equator. We may adapt formula (4) to the discussion of this temperature, interpreting now ϕ as the latitude and Ω as the angular velocity of the sun. We may then introduce

$$ds = +Rd\phi ,$$

R being the radius of the sun, and write

$$\frac{dv}{dz} = \frac{v}{h} ,$$

h being the thickness of the overlying current of the upper circulating stratum. Writing further

$$v_E = \Omega R$$

for the circumferential velocity at the sun's equator, we find from formula (4)

$$\frac{d\vartheta}{\vartheta} = \frac{2v_E}{gh} v \cos \phi d\phi .$$

This must now be integrated from the equator to the pole, i.e., from $\phi = 0$ to $\phi = \pi/2$. Writing the first member in the approximate form $\Delta\vartheta/\vartheta$, we obtain

$$\frac{\Delta\vartheta}{\vartheta} = \frac{2v_E v}{gh} .$$

For comparison we may first apply the formula to the earth's atmosphere. Here the height of the circulating stratum is roughly

10,000 meters, and for the circumferential velocity of the equator we may write $v_E = 500$ m/sec. Using the average temperature $\vartheta = 273°$, we obtain

$$\Delta\vartheta = 2.73\, v\,,$$

v representing the eastward velocity of the highest strata of the troposphere, that at the ground for simplicity being taken as zero. The formula gives

$$\text{for } v = 10 \text{ m/sec.} \qquad \Delta\vartheta = 27° \;;$$
$$\text{for } v = 20 \text{ m/sec.} \qquad \Delta\vartheta = 54° \,.$$

Thus a velocity of 10 m/sec. in the upper part of the troposphere is sufficient to maintain a temperature-difference of 27° between pole and equator, while the double velocity of 20 m/sec. corresponds to the double temperature difference of 54°. The actual difference, when we disregard the thin locally cooled stratum at the surface in the polar regions, is somewhere between 30° and 40°, corresponding to a velocity in the upper troposphere between 11 and 15 m/sec.

Applying the same formula to the sun, we have the circumferential velocity at the equator $v = 2000$ m/sec., the acceleration of gravity $g = 275$, and for the eastward velocity of the highest circulating stratum the value just found, $v = 130$ m/sec., while the thickness h of this stratum remains unknown. Setting $\vartheta = 6000°$ for the surface temperature of the sun, we find for the temperature difference between pole and equator

$$\Delta\vartheta = \frac{12{,}000{,}000}{h}\,.$$

This gives

$$\text{for } h = \;\; 10 \text{ km}, \qquad \Delta\vartheta = 1200° \;;$$
$$\text{for } h = \;\; 100 \text{ km}, \qquad \Delta\vartheta = 120° \;;$$
$$\text{for } h = 1000 \text{ km}, \qquad \Delta\vartheta = 12° \,.$$

If we venture a guess harmonizing with the conjectured thickness of about 100 km for sun-spot vortices, something between 100 km and 1000 km would be the most probable depth for the solar eastward current. The temperature difference of 12° to 120° between

pole and equator would then be of the same order as that on the earth, and probably difficult to detect, and still more so when we remember that these temperatures correspond to the balanced motion, in which we neglect the disturbance by the radiation. But for a greater internal stability of the sun, the depth of sun-spot vortices and of the individual layers participating in the stratified circulation would decrease. Thus, greater temperature differences are quite possible, and actual measurement of such a difference between pole and equator would be of great interest.

Finally, it may be remarked that the circulation assumed for the photosphere should, by frictional coupling, give the solar atmosphere a circulation like that of the earth's atmosphere, i.e., from the poles to the equator in the lower strata, and from equator to poles in the high layers. Under these conditions the high masses should show acceleration relative to those below, limited by friction against and mixing with these lower masses. As a consequence of balance between accelerating and retarding effects, a rotation practically equal to that of the equator might occur for the high strata, in accordance with the result found for the highest parts of the sun's atmosphere.

To the attempts here made to explain the equatorial acceleration of the sun as a result of internal circulations, an important remark must be added: The planet Jupiter shows the same equatorial acceleration. The question must remain open whether the internal constitution of this planet is such as to allow the same explanation as suggested for the sun.

19. *The sun-spot cycle and the reversal of polarities.*—The preceding considerations have led to an interpretation of both sun-spots and equatorial acceleration as phenomena related to the internal circulation of the sun. Sun-spots depend upon intensive local circulations, and the equatorial acceleration upon the slow general circulation. The remaining question is whether these two suggested explanations fit into each other in such a way as to explain the fact that sun-spot phenomena are restricted to the lower latitudes of the sun and have their characteristic periodicity.

Figure 11 shows that we meet with the greatest contrast of temperature on both sides of the equator where the cooled masses of the photosphere, which have been radiating since they left the

poles, descend and come in contact with the hot ascending masses of the next lower circulating layer. This provides conditions favorable, on the one hand, for a concentration of the general circulation in the lower latitudes, which is indicated by an inner circulating curve in each stratum of Figure 11 and, on the other, for the formation of the extra vortices of zonal character to which we have referred the sun-spots. If these zonal vortices have a certain persistency, they

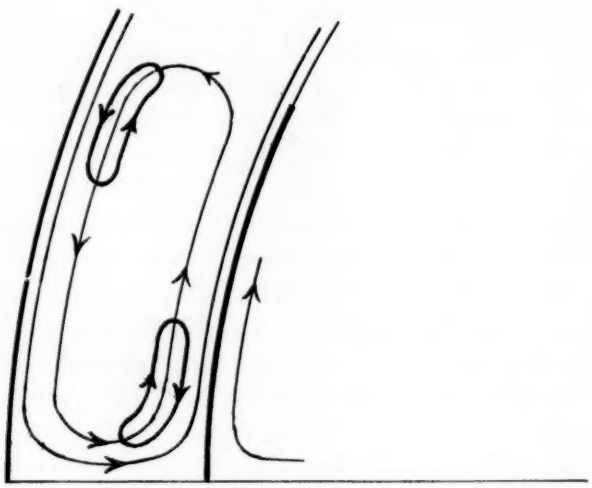

FIG. 12

will be carried round by the general circulation into the lower latitudes.

It is only necessary to admit the existence of two zonal vortices having opposite directions of rotation which are carried round by the general circulation (Fig. 12). From a thermal point of view this supposition is possible, inasmuch as both directions of rotation are compatible with the thermal origin of the vortex zones. And, mechanically, vortices rotating like toothed wheels assist each other by their mutual frictional effect. Since we have estimated that the circulation between the poles and the equator requires many years, a period of twenty-two years for the revolution of the zonal vortices about each other does not seem unreasonable. The succession of the phenomena will be just as observed: The new cycle, with opposite

rotation and opposite polarity, will begin in the higher latitudes while the old cycle is disappearing at the equator. When the zonal vortex at the equator dives below, we shall have only spots of the succeeding cycle which has then come to full development. Then, as this zonal vortex approaches the equator, the other vortex rises in higher latitudes and produces the spots of the next cycle with their reversed polarities, and so on.

From a kinematical point of view the process is exceedingly simple; but we have made only the very first steps in the discussion of its dynamics and thermodynamics.

The frictional coupling of the circulations of the two hemispheres may be responsible for the fact that sun-spots in the two hemispheres have the same periodicity and show opposite magnetic polarities in any given cycle.

The theory thus developed seems to favor the vortical nature of sun-spots. The vortex theory leads to a simple explanation of the low temperatures of sun-spots and gives, as a matter of course, the explanation of the fact that the low temperatures are found in depressions on the sun's surface. It provides conditions under which the low temperatures in these depressions may be maintained for any length of time, and suggests connections between sun-spots which would explain their striking collective properties, including their periodicity and the remarkable reversal of their magnetic polarities. It is not to be supposed that the complex of suggestions here tentatively developed will remain unchanged. Weak points are obvious and others may turn up; but many modifications of the suggested scheme are possible. That proposed will have done its service if it prepares the way for a better one.

BERGEN, NORWAY
November 1925

THE CONTINUOUS SPECTRUM OF HYDROGEN

By IRA M. FREEMAN

ABSTRACT

A continuous spectrum of hydrogen, extending from the yellow-green region far into the ultra-violet, is described; and the method of excitation is set forth.

The discharge tube used was equipped with a hot, coated cathode. Alterations in the spectral character of the glow with changes in the electrical conditions in the tube are indicated.

Intensity measurements performed on the continuous glow reveal a maximum of spectral energy between 4800 and 5100 A.

Introductory.—The existence of a continuous hydrogen spectrum extending from the yellow-green region down to the limit of transmissibility of quartz in the ultra-violet has been noted by several investigators, who have attempted to excite this radiation by a number of different methods.

In his studies on the spectra of the planetary nebulae, W. H. Wright[1] has observed a continuous spectrum of hydrogen which seemed to have similar characteristics in all of the large number of nebulae examined.

It was observed by Horton and Davies,[2] in their researches on critical electron velocities in hydrogen, that slow-moving electrons gave rise to a continuous radiation.

H. B. Lemon,[3] using a discharge tube with a Wehnelt cathode, succeeded in producing in succession, and almost pure, the Balmer lines, the secondary spectrum, and the continuous spectrum by varying the filament temperature, and found this could be accomplished also by varying the potential between the filament and grid or plate.

O. W. Richardson and T. Tanaka,[4] also employing hot cathodes, observed the discharge described by Lemon, and recorded similar conditions for the successive appearance of the three types of spectra.

[1] *Publications of the Lick Observatory*, **13**, 1918.

[2] *Philosophical Magazine*, **46**, 892, 1923.

[3] *Nature*, **113**, 127, 1924.

[4] *Nature*, **113**, 192, 1924; *Proceedings of the Royal Society*, A, **106**, Dec. 1, 1924.

Using hydrogen and nitrogen canal rays in hydrogen, A. Carst[1] repeated experiments originally undertaken by Stark, and attempted to measure, photographically, the intensity distribution of the continuous radiation in the ultra-violet; but because a knowledge of the plate sensitivity was lacking, the measurements could hardly be dependable. He did conclude definitely, however, that the hydrogen molecule is the carrier of this spectrum, as did also Horton and Davies (*loc. cit.*), and that it is entirely distinct from the continuous spectrum which is to be expected at the series limit.

The principal facts given by the foregoing workers have been corroborated by Gehrcke and Lau;[2] and quite recently Schuler and Wolf[3] have noted the apparent independence of the intensity of the continuous- and the series- and secondary spectra. They conclude, also, that the source of the continuous glow is the recombination ("*wiedervereinigung*") $H+H \rightarrow H_2$.

The points agreed upon by most of the workers in this field may be stated as follows: (1) The spectrum begins in the visible region and extends far into the ultra-violet. (2) It is excited by slow electrons, and at rather high pressures. (3) It is distinct from the Bohr spectrum encountered at the series limit.

The present work concerns the examination of the conditions governing the appearance of the continuous glow, and involves the measurement of its spectral energy distribution in the visible region.

Description of apparatus.—The general design of the apparatus is shown in Figure 1*a*. The hydrogen, liberated electrically from acidulated water in the large generator *A*, is dried by passage through anhydrous $CaCl_2$ and P_2O_5 in the drying tubes *B*. The capillary *C* is inserted in the line to furnish resistance to the pump, thus permitting gas to flow continuously. The large-bore by-pass *D* may be opened when a rapid flow of hydrogen is desired, e.g., in flushing out the line. The spiral tube *E* gives flexibility to the tubing and permits small adjustments in the level of the tube before the instrument.

The discharge tube itself (*F*) is a modification of the hot-cathode tube used by Lemon. The braided coated filament[4] is about

[1] *Annalen der Physik*, **75**, 665, 1924. [2] *Ibid.*, **76**, 673, 1925.

[3] *Zeitschrift für Physik*, **33**, 42, 1925.

[4] Supplied through the courtesy of the American Telephone and Telegraph Company.

10 cm from a cylindrical metal anode. The observing instrument is placed before the window *W*, which is constructed so that a view of the space just in front of the filament is obtainable.

The function of the bulb *R* is to provide a dark background, free from troublesome reflected light. The entire inner surface of the bulb is ground, so that there is very little chance of light being reflected from its interior. This device, it is believed, constitutes an

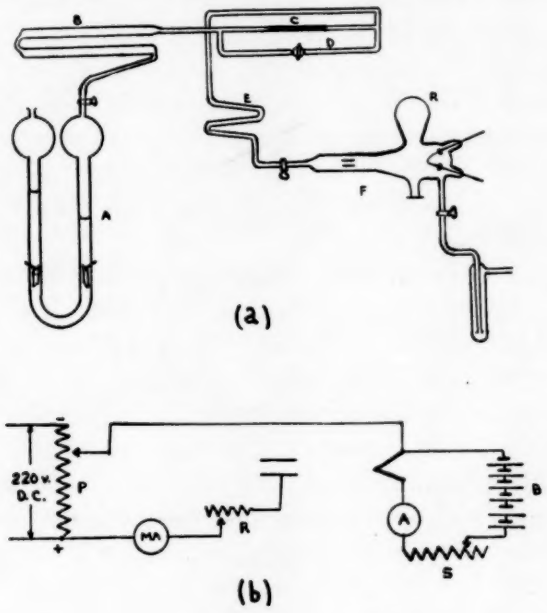

FIG. 1

ideal background, for it does not involve the placing of screens or black paint of questionable composition inside the tube. The idea of a "dark cave" background is probably due to Rayleigh, who used such an arrangement in his work on the scattering of light.

The tube is provided with a stopcock at each end. The line next leads to a liquid-air condensing trap, and thence to an electrically driven oil pump of the eccentric type.

The electrical connections to the tube are shown diagrammatically in Figure 1*b*. A potentiometer *P* permits the application of all or a part of the full voltage, while a series resistance *R* controls

the current through the tube. A 12-volt storage battery *B* heats the filament.

Experimental procedure.—When the pressure within the tube has been reduced to 1 or 2 mm, the glow in the positive column of the tube consists of pink, bell-shaped striations extending from the anode as far as the observing window. There is also a glow around the cathode which will now be described: With the filament current at a low value and the potential across the tube a little less than 150 volts, the cathode glow exhibits the Balmer series—almost alone except for a weak luminous background. A small increase in the heating current now brings forth the secondary spectrum, which gains in intensity at the expense of the series lines—now totally extinct.

A further increase in the filament current causes the slight pink aura around the filament to give way suddenly to a brilliant blue. This blue glow shows no trace of either the series or secondary lines, and is continuous when viewed in a direct-vision instrument. It frequently happens that the first striation in the positive column is also blue, and when this occurs, it is spectrally similar to the glow at the cathode.

In order to rid the spectrum of the Ångstrom (*CO*) bands which were present, because of impurities, the pump was made to draw gas continuously through the apparatus while the observations were in progress. This served to eliminate the objectionable bands completely.

During all the runs, the filament was heated with a 10-ampere current, and a potential difference of about 211 volts was put across the tube. The ionization current was approximately 42 milliamperes.

Conditions in the tube were found to be reproducible at different times with remarkable ease. The guide was the number and appearance of the hyperboloid striations extending from the anode.

The photometric observations were made with Lemon's modification of the Brace spectrophotometer.[1] After preliminary adjustments, the instrument was calibrated, using the lines of mercury and

[1] H. B. Lemon, "A Polarization Spectrophotometer Using the Brace Prism" *Astrophysical Journal*, **39**, 204, 1914.

of helium. This furnished a dispersion curve giving the wave-lengths corresponding to readings on the arbitrary vernier scale on the telescope of the instrument.

The continuous spectrum was compared with the radiation from a tungsten filament lamp at several points in the spectrum. Five settings of the polarizing apparatus were made at each point, the intensity match being approached alternately from the darker and from the lighter sides. In order that the spectrum of the filament in the tube should not be superposed on that of the hydrogen, the instrument was so directed that cutting off the electric field, and leaving the filament glowing, caused the field of view to become totally dark.

The substandard lamp used above was then placed before the other collimator of the spectrophotometer, and was compared with a standard lamp of known color temperature. In this way the absorption in the instrument is taken into account.

Knowing the color temperature of the standard lamp at the voltage used (80 volts), its energy distribution was computed from Wien's law:

$$I = k\lambda^{-5} e^{-\frac{c_2}{\lambda T}},$$

using Coblentz' value of c_2: 14,350 micron degrees.[1]

This curve is shown in Figure 2*a;* the curve of the substandard lamp is shown to the same scale. The latter was obtained, of course, by dividing the ordinates of the standard lamp curve by $\sin^2 \theta$ where θ is the complement of the angle between the transmitting planes of the nicols when an intensity match is attained.

The observations on the spectrum, taken during four different runs, were graphically averaged; the nicol settings were all found to lie close to a curve which was then taken as the most probable relation of wave-length and intensity ratio in the spectra of the lamp and the continuous radiation of hydrogen. This is shown in Figure 2*b*.

The actual distribution in the hydrogen spectrum was then obtained by multiplying the ordinates of the substandard lamp curve by $\sin^2 \theta$—each value of θ being obtained from Figure 2*b*. This dis-

[1] W. W. Coblentz, *Scientific Papers of the U.S. Bureau of Standards*, No. 284, 1916.

tribution is shown in Figure 2*a*, to a different scale than the other two curves. It rises quite steeply to its largest values between 4800 and 5100 A, declining in energy rather gradually toward the shorter wave-lengths.

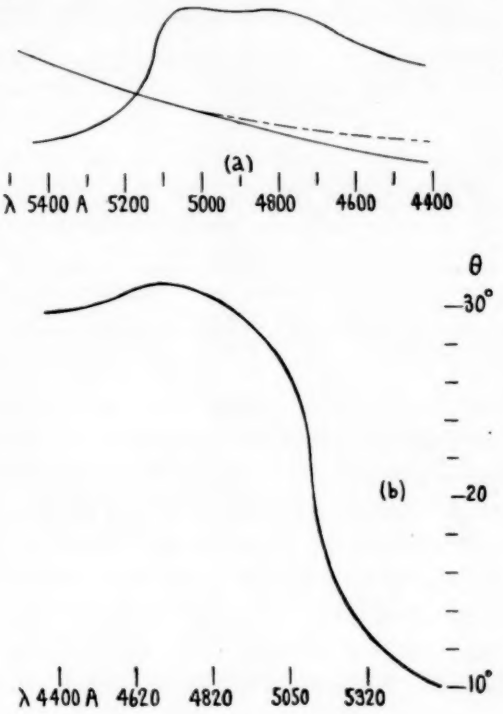

FIG. 2

Conclusion.—The present investigation shows that the excitation of the continuous spectrum of hydrogen in tubes employing hot activated filaments is reliable and readily reproducible in all details.

The greater part of the energy of the spectrum, in the visible region, was found to be in the green and blue portions.

Acknowledgment is due Dr. H. B. Lemon for suggesting the problem, and for continued personal assistance during the course of the work.

RYERSON PHYSICAL LABORATORY

PHOTO-ELECTRIC PROPERTIES OF THIN FILMS OF ALKALI METAL

II. PHENOMENA AT HIGH TEMPERATURES

By HERBERT E. IVES

ABSTRACT

A *platinum ribbon in an atmosphere of caesium vapor* is heated to various temperatures up to incandescence by the passage of an electric current. By keeping the vapor pressure of the caesium low by cooling the walls of the tube, thermionic currents and photo-electric currents, caused by illuminating the ribbon, are obtained of the same order of magnitude, and their variation with temperature determined. Both *thermionic and photo-electric currents increase with temperature to a sharp maximum and then decrease to negligible values.* Consideration of the relative magnitudes of the two currents leads to the conclusion that the *thermionically emitted electrons cannot be due to an internal photo-electric excitation.*

Introduction.—In an earlier paper[1] I have described some photo-electric phenomena exhibited by thin films of alkali metal as spontaneously accumulated on polished metal surfaces in a high vacuum. In those experiments, the film was reduced in thickness from its normal equilibrium value, reached with the entire cell at room temperatures, by heating the underlying metal plate through electronic bombardment and capturing the evaporating alkali metal on the cold glass walls of the cell. Observations were made either after the metal plate had completely cooled, or, in some cases, during the course of its cooling. In general, it was found that as the alkali metal film increased in thickness from zero, the photo-electric effect at first increased, reached a pronounced maximum, and then declined. Variations of the photo-electric effect with wave-length and with plane of polarization of the incident light were followed in some detail.

The experiments to be described here also have to do with thin films, but the circumstances of their existence are substantially different. The experimental procedure consists in maintaining the metal plate at a definite high temperature by the passage of a heavy electric current. Whereas in the former experiments no measure-

[1] "Photo-electric Properties of Thin Films of Alkali Metals," *Astrophysical Journal*, **60**, 209, 1924.

ments were made while the plate was still incandescent after heating, or in the temperature range where thermionic emission is present, these later experiments extend the observations of the photo-electric effect to incandescence temperatures of the metal plate, and measurements are made simultaneously of thermionic and photo-electric emission.

Apparatus.—The photo-electric cells used in this study are of the type shown in Figure 1. The metal plate on which the alkali metal was allowed to accumulate is in the form of a platinum ribbon S, 2 mil thick, of surface dimensions $1\frac{1}{4}$ by $\frac{3}{16}$ inches. This is held between heavy nickel clamping pieces N, one of which is rigidly attached to a heavy leading-in wire L_1, the other is attached to a second leading-in wire L_2 by a flexible connection F. The platinum ribbon is kept straight by tension of a spiral spring P. Heating of the ribbon is accomplished by the passage of a heavy alternating current taken from the secondary of a transformer, the primary of which is operated from a 110-volt supply. The anode of the photo-electric cell consists of a large cylinder of sheet nickel A, in which a rectangular opening extending through 180° is cut. A charcoal tube T is provided for immersion in liquid air, and a side bulb C contains a small quantity of alkali metal, which is introduced into it by distillation, while the tube is connected to the pump, and is later driven into the main body of the cell. The cell is made of pyrex glass, and the method of baking out and evacuating is the same as that described in earlier papers.

It was hoped to study the photo-electric effect for different planes of polarization of the incident light and at individual wave-lengths. This, however, proved impracticable for several reasons. The measurements of the photo-electric effect for different planes of polarization of the incident light were prevented by the fact that the platinum strip when heated by heavy alternating current developed a rough crystalline surface, so that the ratio of photo-electric emission for the two planes of polarization never reached a high value. Measurements at different wave-lengths were rendered exceedingly difficult because of the very intense illumination necessary to secure a photo-electric current comparable in size with the thermionic current which occurred at the higher temperatures.

The illumination was effected in all cases described by focusing an image of a helical tungsten filament directly on the platinum

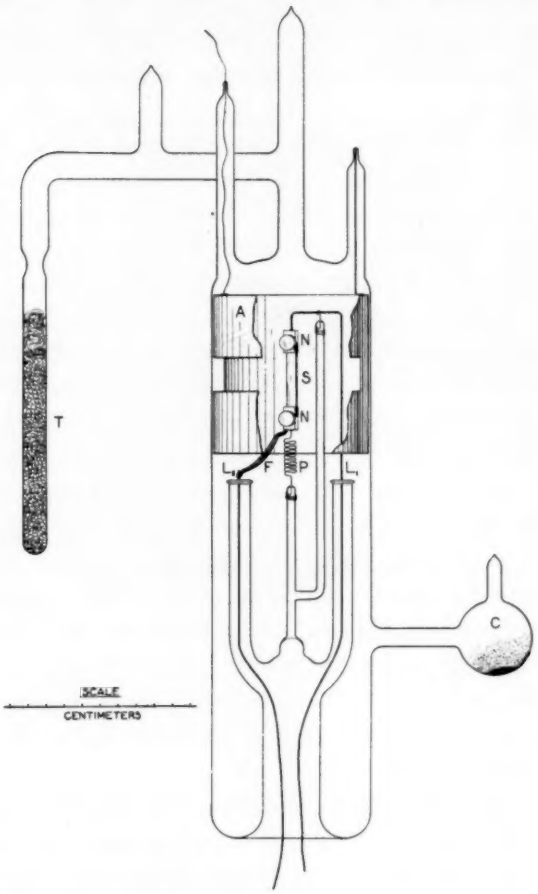

FIG. 1.—Photo-electric cell used for studying photo-electric and thermionic properties of alkali metal films on heated platinum.

S—Thin platinum ribbon
N—Clamps for holding ribbon
P—Spring to hold ribbon taut
F—Flexible current lead
L_1 and L_2—Copper leading-in wires
A—Nickel cylinder anode
T—Charcoal tube
C—Alkali metal

ribbon, the light being incident at about 60°. Currents were measured by a D'Arsonval galvanometer fitted with shunts to vary the sensibility.

Temperatures of the platinum ribbon were measured by means of an optical pyrometer with disappearing filament, corrections being made for the emissive power of the platinum and for the light lost by the interposition of the glass wall of the cell and a lens, which was used to form an image of the ribbon in the focal plane of the pyrometer eyepiece. The curve for temperature versus heating current, thus obtained, was extrapolated below the optical range by determining constants representing losses of radiation and conduction over the range of calibration of the pyrometer, and by using these same constants to extend the curve downward.

Measurements.—The experimental procedure with small heating currents through the platinum ribbon was perfectly straightforward. After the heating current had been on sufficiently long for conditions of equilibrium to be attained, the ribbon was illuminated and the photo-electric current determined. As the heating currents increased, however, a number of experimental difficulties were encountered. The large masses of metal in the ribbon clamps and leading-in wires radiated large amounts of heat, and the whole tube became quite hot. This raised the vapor pressure of the alkali metal, and caused the thermionic current due to it to become so large that such photo-electric current as could be produced by illuminating the heated ribbon was considerably smaller than the fluctuations of the thermionic current caused by unavoidable irregularities in the current heating the ribbon. In later cells the size of the platinum ribbon was reduced to a minimum and the masses of other parts kept down, but these expedients proved inadequate. The procedure finally adopted was indicated by the results of a study of the thermionic emission from thin films of alkali metal reported elsewhere.[1] From this study it appeared that the thermionic emission from the alkali metal, which occurs only over a narrow temperature range, may be greatly reduced by holding down the vapor pressure. Accordingly, trials were made of the effect of keeping the walls of the cell cold. The procedure finally adopted was to use caesium as the alkali metal, and to pack around the glass walls of the cell a thick layer of carbon dioxide snow. When this is done the thermionic

[1] "Positive Rays Produced in Thermionic Vacuum Tubes Containing Alkali-Metal Vapors," *Journal of the Franklin Institute*, **201**, 47, 1926.

emission may be reduced to the same order of magnitude as the photo-electric emission, so that both may be measured on the same galvanometer. The experimental procedure consists in measuring first the thermionic current and then the total current when the platinum strip is illuminated. The difference between these two currents is the photo-electric current.

In Figure 2 are shown typical results obtained under the conditions of illumination described and for ribbon temperatures as indi-

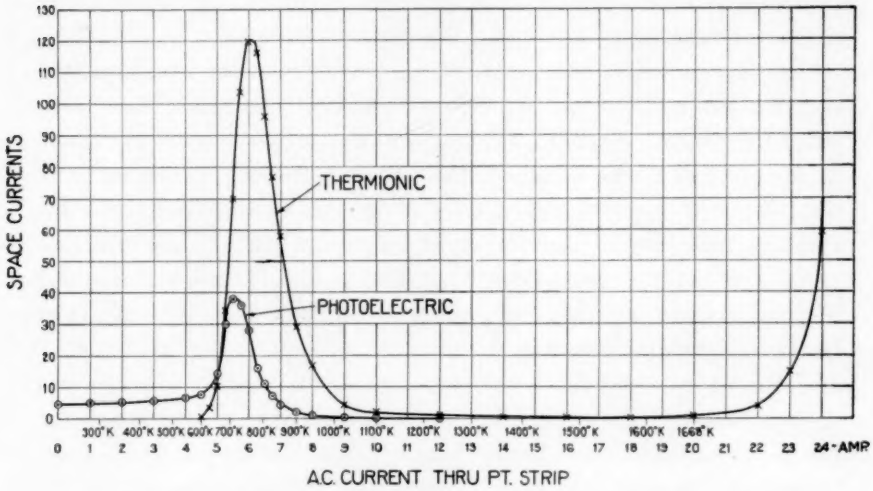
SPACE CURRENTS
THERMIONIC
PHOTOELECTRIC
300°K 400°K 500°K 600°K 700°K 800°K 900°K 1000°K 1100°K 1200°K 1300°K 1400°K 1500°K 1600°K 1668°K
A.C. CURRENT THRU PT. STRIP
24-AMP

FIG. 2.—Photo-electric and thermionic currents from platinum strip in low-pressure caesium vapor. Abscissae—(upper) temperatures of platinum strip; (lower) A.C. currents through platinum strip.

cated. The thermionic emission follows the general course with temperature previously described elsewhere[1]. The emission from the alkali metal starts in at a comparatively low temperature, increases rapidly, passes through a maximum, and decreases to practically zero value, this trend being due presumably to the inability of the alkali metal to adhere to the incandescent surface. A region of negligible thermionic emission follows, after which the thermionic emission of the platinum surface begins. The photo-electric emission changes but little over a considerable range of temperature, increasing, however, slightly with rising temperature. At approximately

[1] *Journal of the Franklin Institute*, **201**, 59, 1926.

650° K., the photo-electric emission increases very rapidly, rising to a peak of value five to ten times that at room temperature. As the temperature of the ribbon increases, the photo-electric current decreases again, becoming inappreciable at about 1200° K.[1]

The data shown are of the general type obtained in several sets of measurements. Owing to the very complicated conditions, which preclude accurate control of temperature, the curves representing different experimental runs differ somewhat, both in the magnitudes of the effects and in the relative values of thermionic and photo-electric emission. In the curve shown the successive points were taken with increasing temperatures of the strip. When observations are made with decreasing heating current, the maxima of both thermionic and photo-electric emission shift to somewhat lower values of heating current and their absolute and relative magnitudes are somewhat changed. Until, however, much more accurate control of conditions has been attained than has thus far been possible, it does not appear worth while to give a detailed account of the minor departures from the primary phenomena which are well exhibited in the diagram.

Discussion.—So far as I am aware this is the first time it has been possible to obtain overlapping measurements of thermionic and photo-electric emission. In general, it may be said that each kind of emission behaves in the way one would anticipate from previous studies on photo-electric and thermionic emission separately. The enhancement of photo-electric emission at the higher temperatures may be due to an increase in photo-electric emissive power with temperature, but I am inclined to the belief that, like previously effected variations of photo-electric current with thickness of film, it is due solely to the diminishing quantity of photo-sensitive material per unit surface area. The rise and fall of emission with the temperature of the platinum strip is in fact very much like the cycle of variations shown in Figure 10 of the paper previously referred to,[2] where the film thickness was decreased by successive glowings of the underly-

[1] Comparison of Fig. 2 with the figure illustrating the paper on positive rays (p. 59) shows that the photo-electric effect becomes inappreciable at about the same temperature that the positive current begins. Careful tests made in the course of the present experiments failed to show any influence of illumination on the positive current.

[2] *Astrophysical Journal*, **60**, 222, 1924.

ing surface. The relative positions of the thermionic and photo-electric maxima, the former always occurring at higher temperatures than the latter, are in agreement with this conception, because the enhancement of thermionic emission with temperature is so extremely rapid as to offset the effect of diminishing material much longer than can the relatively small magnification of the photo-electric effect which occurs on attenuating the layer of the alkali metal.

The simultaneous occurrence of photo-electric and thermionic emission is of interest in connection with a problem which has been discussed by O. W. Richardson,[1] namely, the possibility of thermionic emission being actually a photo-electric emission induced by the radiation from the glowing metal. In the experiments here described, it is worthy of note that the platinum strip was emitting visible radiation at a temperature of 800° K. As earlier experiments have shown,[2] the alkali metals are more photo-sensitive in attenuated films than in massive blocks, and this extra sensitiveness is accompanied by an extension of the long-wave limit, which in the case of caesium is pushed well into the infra-red. Consequently, at the higher temperatures the photo-electric emission was certainly to some degree induced by the radiation from the underlying platinum ribbon.

It is of course true that the flux density of radiation in the image of the very bright external light-source focused on the platinum ribbon is much greater than that due to the barely glowing platinum ribbon itself. Recent experiments on the photo-electric effect of the alkali metals all indicate,[3] however, that photo-electric emission is an extremely superficial phenomenon, when excited by external illumination. Thermionic emission, on the other hand, probably occurs from considerable depths of the material. It is consequently of some interest to see that under the conditions of the experiments here described it has been possible, by decreasing the volume of material, to bring the two kinds of emission at least to the same order of magnitude.

[1] See *The Emission of Electricity from Hot Bodies*, 2d ed.

[2] *Astrophysical Journal*, **60**, 216, 1924.

[3] "The Electrical and Photo-electric Properties of Thin Films of Rubidium on Glass," *Astrophysical Journal*, **62**, 309, 1925.

The possibility of a still closer approximation in value of the two emissions, or their complete identification under any conditions likely to hold within the body of the material, seems, however, on the whole, to be rendered improbable rather than probable by these experiments. Thus it is reasonably certain that at some temperature of the platinum strip the caesium is present as a monatomic layer. Every caesium atom will be a "surface" atom, so that the maximum photo-electric emission which radiation from surrounding matter could excite would actually occur; there would be no atoms capable of emitting thermionically which were not equally free to emit photo-electrically. It should therefore be possible at some temperature to obtain a photo-electric current equal to the original thermionic current, by illuminating the platinum ribbon with an image which, when formed upon a white matte surface, would appear no brighter than the glowing ribbon. This would correspond to a very small fraction of the radiant flux density used in any of the experiments here described, in none of which did the photo-electric emission ever equal the thermionic. Therefore, unless it develops that atoms in the interior of a radiating body are much more favorably situated for photo-electric excitation than the layer of surface atoms just considered, it would appear that these experiments support the view that thermionic and photo-electric electrons are liberated by essentially different agencies.

Acknowledgment.—I am indebted to Mr. A. L. Johnsrud and Mr. A. R. Olpin for carrying out the difficult manipulations and measurements which form the subject matter of this paper.

BELL TELEPHONE LABORATORIES, INCORPORATED
NEW YORK CITY
March 3, 1926

ON THE THEORY OF THE CONTINUOUS SPECTRUM OF THE CORONA[1]

By EDISON PETTIT and SETH B. NICHOLSON

ABSTRACT

The distribution of energy in the coronal spectrum.—Woltjer's calculations, based on scattering by free electrons, indicate that the ratio of far infra-red to visual radiation is greater for the corona than for the sun. A recalculation without his approximations gives an even richer infra-red radiation for the corona.

Computation of the water-cell transmission of the coronal radiation.—It is shown that, even with the revised calculations, the difference between the spectral distributions of energy in the coronal and solar radiations could not be detected by using the water cell to separate two spectral regions, as was done at the eclipse of January 24, 1925.

Agreement of computed and observed water-cell transmissions.—The theoretical water-cell transmission for the corona agrees as closely as can be expected with that observed by us at the eclipse of January 24, 1925.

In a recent paper, Woltjer[2] has calculated the distribution of energy in the continuous spectrum of the corona based on the scattering of radiation by free electrons. Two approximations are given, the first depending on the classical theory of scattering, and the second, which he accepted as more conclusive, depending on the introduction of the theory of quanta. The effect of positive ions is referred to, but their influence and that of neutral atoms is not discussed; hence the result tacitly assumes the corona to be an atmosphere of free electrons. The conclusion is that, while the continuous spectrum of the corona is practically identical in energy distribution with the solar spectrum in the photographic and visual region, it is much richer in energy in the far infra-red. It is our purpose to discuss the bearing of the radiometric observations on the author's theory.

In the approximations based on the classical theory of scattering Woltjer gives the following expression for the radiation scattered toward the earth from a point P in the corona:

$$2\pi r_0^2 \int_0^{\psi_0} \frac{I_\nu}{r^2} \cos\theta \sin\psi (1+\cos^2\chi \cos^2\alpha+\tfrac{1}{2}\sin^2\chi \sin^2\alpha) d\psi . \qquad (1)$$

To integrate this expression, he neglected the higher powers of χ and arrived at the approximate solution

[1] *Contributions from the Mount Wilson Observatory*, No. 314.

[2] *Bulletin of the Astronomical Institute of the Netherlands*, **3**, 103, 1926.

$$F_\nu\left(1+\cos^2 a+\tfrac{1}{16}\frac{r_0^2}{\Delta^2}\sin^2 a\,\frac{15a_\nu+8b_\nu}{3a_\nu+2b_\nu}\right), \qquad (2)$$

where F_ν measures the energy of frequency ν in the mean solar spectrum, a_ν and b_ν are the coefficients of darkening for the solar atmosphere, and the relations

$$\frac{r_0}{\Delta}=1, \qquad a=\frac{\pi}{2}$$

hold when the point P is at the limb and on the sun.

If the higher powers of χ are not neglected, the solution for a point at the limb and on the sun is

$$F_\nu\left(1+\tfrac{1}{4}\frac{20a_\nu+7b_\nu}{3a_\nu+2b_\nu}\right). \qquad (3)$$

The coefficient of F_ν is the ratio of intensity of the coronal radiation to that of the integrated solar radiation for any frequency ν. Table I gives the values of this ratio derived from Woltjer's expression (2) and from the rigorous expression (3) in units of their values for wave-length 0.3 μ.

TABLE I

Wave-Length	Approximate (2)	Rigorous (3)
0".3	1.00	1.00
0.6	1.03	1.15
1	1.04	1.23
2	1.05	1.27
3	1.05	1.27
4	1.05	1.28
5	1.05	1.28

It will be noted that the rigorous formula (3) indicates more infra-red radiation in the corona than formula (2).

Introducing the quantum relations, the author gives the coefficient of scattering, according to Pauli[1] and others, as proportional to

$$1+\frac{c^3}{8\pi h\nu^3}\rho_\nu, \qquad (4)$$

[1] *Zeitschrift für Physik*, 18, 283, 1923.

where the frequency of the incident and scattered radiation is taken to be the same, and the function is supposed to be roughly applicable to non-isotropic radiation.

In this expression, ρ_ν is the density of radiation at the point P, and is equal to

$$2\pi \frac{r_0^2}{c}\int_0^{\psi_0} \frac{I_\nu}{r^2} \cos\theta \sin\psi \, d\psi \, . \tag{5}$$

In the integration of (5) the higher powers of χ were again neglected with the following result:

$$\rho_\nu = \frac{F_\nu}{c}\left(1+\tfrac{1}{20}\frac{r_0^2}{\Delta^2}\frac{15a_\nu+8b_\nu}{3a_\nu+2b_\nu}\right) = \frac{F_\nu}{c}(1+\beta) \, . \tag{6}$$

Substituting in (4) the value of ρ_ν from (6) and taking

$$F_\nu = \frac{r_0^2}{\Delta^2}\frac{2\pi h\nu^3}{c}\frac{1}{e^{\left(\frac{h\nu}{kT_e}\right)}-1},$$

we find for the factor which expresses the dependence of the coefficient of scattering on ν

$$1+\tfrac{1}{4}\frac{r_0^2}{\Delta^2}\frac{1+\beta}{e^{\left(\frac{h\nu}{kT_e}\right)}-1} \, . \tag{4a}$$

In evaluating this coefficient Woltjer neglected β. Integration of equation (5) without approximations gives for a point at the limb and on the sun

$$\rho_\nu = \frac{F_\nu}{c}\left(1+\frac{3a_\nu+b_\nu}{3a_\nu+2b_\nu}\right) = \frac{F_\nu}{c}(1+\beta_1) \, . \tag{6a}$$

Table II shows the factors of proportionality (4*a*) as derived by Woltjer, neglecting β, and as derived from equation (6*a*), including the value of β_1.

Here again the rigorous formula indicates more infra-red radiation than the approximate formula.

In discussing the observational data it will be recalled that the distribution of energy in the coronal spectrum was determined radiometrically at the eclipse of January 24, 1925, by measuring the water-cell transmission.[1] The transmission constants of this cell have

[1] *Mt. Wilson Contr.*, No. 299; *Astrophysical Journal*, **62**, 128 and 327, 1925.

already been given.[1] The distribution of coronal radiation under the conditions of observation may be computed by applying the theoretical factors given in Tables I and II to the intensities in the solar spectrum after transmission by the earth's atmosphere, and then applying the water-cell transmission constants to the resultant intensities. The data in Tables I and II refer to a point on the sun at the limb, and with less approximation to other points in the corona situated in the same line of sight. Since the intensity in the corona falls off rapidly with increasing distance from the limb, it is clear that points which approximate the conditions for which Tables I and II were computed will have the predominating effect; but these limi-

TABLE II

Wave-Length	Approximate (2)	Rigorous (3)
0$^{\mu}$.3	1.00	1.00
0.6	1.00	1.01
1	1.02	1.04
2	1.10	1.19
3	1.20	1.38
4	1.30	1.58
5	1.40	1.78

tations should be kept in mind when comparing the theoretical and observational data.

If we could place equal confidence in the theories from which the values in Tables I and II are derived, the combined result, represented by the products of the two solutions, would be adopted as the more reliable; but since it is difficult to estimate the effect of several rather violent assumptions made in the applications of the theory of quanta, we give, in Table III, two solutions, one depending on the classical theory alone and one depending on the classical theory as modified by Woltjer's application of the quantum theory.

In Table III the numbers in the second column are proportional to the solar energy between the limits of wave-length indicated in the first column. These values, which were taken from Table 550 of the *Smithsonian Physical Tables* (7th rev. ed.), are for air-mass 3, which was traversed by the radiation. The third column gives the products

[1] *Mt. Wilson Contr.*, No. 246; *Astrophysical Journal*, **56**, 295, 1922.

of these values into the transmission coefficients of water-vapor inferred from Table 377 (*ibid.*) for 1 cm of precipitable water. No measures of precipitable water were made at this eclipse; but, considering the observing conditions, 1 cm is a reasonable estimate. A change of 50 per cent in this variable will scarcely affect the argument. The numbers in the third column therefore represent the energy distribution in the solar spectrum under the conditions of observation at the eclipse of January 24, 1925. It is not necessary

TABLE III

Range of Wave-Length	Solar Energy		Coefficient of Proportionality Corona/Sun		Coronal Energy	
	Air-Mass 3 Dry	Air-Mass 3 H_2O, 1 cm	Table I	Tables II and I Combined	Classical	Modified
0.3–0.4μ	0.10	0.10	1.02	1.02	0.10	0.10
0.4–0.5	0.70	0.70	1.06	1.06	0.74	0.74
0.5–0.6	1.00	1.00	1.12	1.13	1.12	1.13
0.6–0.7	0.98	0.98	1.16	1.18	1.14	1.16
0.7–0.8	0.87	0.80	1.18	1.20	0.94	0.96
0.8–0.9	0.72	0.70	1.20	1.23	0.84	0.86
0.9–1.0	0.60	0.50	1.22	1.26	0.61	0.63
1.0–1.1	0.51	0.50	1.23	1.29	0.62	0.64
Sum (0.3–1.1μ)	.....	5.28	.....	.....	6.11	6.22
1.1–2.0	2.43	1.80	1.26	1.41	2.27	2.54
2.0–3.0	0.45	0.20	1.27	1.63	0.25	0.33
3.0–4.0	0.10	0.04	1.27	1.88	0.05	0.08
4.0–5.0	0.05	0.02	1.28	2.15	0.03	0.04
Sum (0.3–5.5μ)	.....	7.34	.....	.....	8.71	9.21
$\frac{0.3\text{–}1.1\mu}{0.3\text{–}5.5\mu}$ = {Water-cell transmission}	.....	0.72	.....	.....	0.70	0.68

to consider the reflection of silver, since it will have the same effect on the coronal and solar radiation, which are identical in the region of selective reflection.

The fourth and fifth columns give the mean values of the coefficients of proportionality between the energies in the coronal and solar spectrum, computed according to the classical theory, and that modified by the quantum theory as outlined by Woltjer. Multiplying intensities of the solar radiation in the third column by these coefficients, we obtain the sixth and seventh columns, which represent the distribution of coronal energy according to these two theories, respectively.

While the water cell transmits some energy to 1.3 μ, its practical limits are 0.3–1.1 μ, the transmission between 1.1 and 1.3 μ being insignificant. The summations of the third, sixth, and seventh columns between the limits 0.3 and 1.1 μ then represent the energies isolated by the water cell, and those between the limits 0.3 and 5 μ, the total energies in the solar spectrum and in the coronal spectrum as computed from theory. This shows that, according to the classical and the combined classical and quantum theories of scattering by free electrons, the water-cell transmissions of coronal radiation should be 70 and 68 per cent, respectively, as against 72 per cent for integrated solar radiation. These small differences are scarcely

TABLE IV

	Source of Radiation	Percentage of Energy 0.3–1.3μ	O–C	
			Classical	Modified
Calculated	Sun	72.	.....	.....
	Corona, classical theory	70.	.....	.....
	Corona, modified theory	68.	.....	.....
Observed	Sun, Pettit and Nicholson	71.4	− 1.	− 1.
	Corona, Pettit and Nicholson	77.6	− 8.	−10.
	Corona, Stetson and Coblentz	33.	+37.	+35.

detectable by a single set of observations made at one eclipse even under the best observing conditions.

A comparison of the observed and calculated values is given in Table IV. The last column gives the differences between the observed and computed percentages of transmission, for both the classical theory and that modified by the theory of quanta as outlined by Woltjer. The first two pairs of values for O − C are probably not beyond the errors of observation, which may have either sign. It will be recalled that our observations with a thin glass screen showed no measurable radiation beyond 5.5 μ in the far infra-red, which is in agreement with the theory when we consider the negligible value of the energy in the solar spectrum in this region.

We wish to express our thanks to Mr. Rosseland for independent solutions and checks on points connected with the theory.

Mount Wilson Observatory
June 1926

REVIEWS

The Theory of Relativity. By L. SILBERSTEIN. Second edition, enlarged. London: Macmillan & Co., Ltd., 1924. Pp. x+563. 25*s* net.

This book is the second edition of Dr. Silberstein's *The Theory of Relativity.* The first edition appeared in 1914, and it contains an outline of the special relativity. The present volume is an extensive development of the first one, and the second part on "General Relativity" deals mainly with the extensions and generalizations made in the theory since 1914. The first ten chapters are taken over from the first edition almost without change, except for minor details; the six remaining chapters constitute the main addition to the first edition. It may be well to recall that, in the first part, the connection had been traced between the theories of the pre-relativistic period and the modern theory of special relativity. The first of the new chapters, being the eleventh of the whole book, gives a good account of the logical transition from the special to the general theory of relativity. So that as a whole, Dr. Silberstein's book gives the interesting historical development of the concepts which form the basis of modern physics. The twelfth chapter is the only one having a thoroughly abstract character; it contains the necessary powerful mathematical instrument: the principles of the tensor calculus, which is at the basis of the theory.

The three following chapters deal, respectively, with "Geodesics as World-Lines of Free Particles and Light," "Gravitational Field Equations and Energy Tensor of Matter," and " Electromagnetism and Gravitation." They may be found in any textbook on relativity. But here we may say that those subjects are probably considered from a different point of view. The treatment of geodesics is a new one, especially that of the light-tracks. An account is here given of the theory of the effect of rotation of the earth on light-waves. We now know that the experiments just performed by Professors Michelson and Gale near Chicago yield results in exact agreement with the theory. We may note here, however, that they may be explained by other theories as well. Mention is also made of the geodesic precession found independently by De Sitter, J. A. Schouten, Kramers, and Fokker. After the treatment of the field equations *in vacuo*, including as a particular case the radial symmetrical field, consideration is given to the crucial phenomena with a discussion of the observational facts.

The theory of the electromagnetic field is given according to Kottler's method, generalizing Minkowski's treatment of the question. As an example, the solution of an electric particle is given. Only slight attention is paid to Weyl's, Eddington's, and Einstein's successive latest generalizations.

The last chapter is concerned with the cosmological aspects of the theory. After having shown how Einstein was led to modify his original field equations by the consideration of the boundary conditions at infinity, Dr. Silberstein gives a thoroughly critical discussion of both Einstein's and De Sitter's solutions. The viewpoint of the former is based on the assumption of a finite elliptic space filled with a world-matter on the average distributed uniformly and at rest. The heavenly bodies should only be a form of condensed world-matter. De Sitter's cosmology is based on the existence of a finite, elliptic, empty space, the heavenly bodies being only isolated singularities. The author here takes the astronomer's viewpoint and considers clearly De Sitter's solution as the most acceptable. He develops the theory on this basis in considerable detail, giving a full account of the spectroscopic distance effect and also of the "total" Doppler effect due at the same time to radial motion and distance. The formulae so obtained seem to be supported by the observational data of remote celestial bodies, namely, the globular clusters for which radial velocities have been obtained by Slipher and others and their distances derived by Shapley. We might possibly object that these considerations can only be taken as pure speculations, although they form, nevertheless, a highly interesting and captivating question.

This general discussion is followed by twelve miscellaneous notes and remarks on different questions, e.g., experimental results concerning the shift of lines in the solar spectrum according to St. John's latest experimental work; further observations on the bending of light-rays around the sun; on world-curvature, and different problems and consequences of a cosmological character. As a matter of clearness and convenience it might have been advisable to subdivide the different chapters into smaller limited sections.

Apart from the development of the fundamental ideas of the relativity theory, the book is probably richer in interesting and valuable details than any other one, at least from the cosmological point of view. Moreover, no efforts have been spared in order to make the physical meaning of the successive statements as clear as possible. Together with the standard treatises of Eddington, De Donder, and Weyl, Dr. Silberstein's book is one which should find a place in every library.

H. L. VANDERLINDEN

Primitive Time-Reckoning. By MARTIN P. NILSSON. Lund: C.W.K. Gleerup; New York: Oxford University Press, 1920. Pp. xiii +384.

This is the tale of primitive peoples and, to some extent, of their descendents, concerning themselves with the manifestations of nature, with particular reference to the use and adaptation of those signs and indications to the purposes of rude record and crude prediction for the needs of their daily life. The genesis of this work lies in the investigation of Professor Nilsson upon the origin and development of the Greek festivals. Brought face to face with questions of chronology, ever widening in their scope and necessitating for the history of their origins many researches into the procedure and customs prevalent in preceding ages, he was led into the domain of the ethnologist, into the method of counting periods of time among the ancient peoples.

The material which the author has discussed was found largely in the works of some 235 authorities, which are listed in a bibliographical index. It is quite apparent, however, that independent and personal research on his part among living sources has supplemented his discussion of the published works of others.

As the basis of time-reckoning there are only a limited number of phenomena, falling into two categories, namely, those of the heavenly bodies, and those of a terrestrial nature—the changing of the seasons and weather and of plant and animal life.

It would, indeed, appear that Professor Nilsson has a very definite intention that the product of his trained and scholarly mind shall be suggestive. He has thrown open the gate into a field in which the careful and industrious student may find much to cull, to analyze, and to co-ordinate.

J. T.